

Number system, Indices, Surds:

Number System

Natural Numbers (N):

$$N = \{1, 2, 3, 4, 5, 6, \dots\}$$

Whole Numbers (W):

$$W = \{0, 1, 2, 3, 4, 5, \dots\}$$

Integers: I = $\{-\infty, \dots, -2, -1, 0, 1, 2, \dots, \infty\}$

Rational Numbers:

- If p and q are integers where $q \neq 0$, then p/q is called a rational number.
- Let us say 'Q' represents the set of rational numbers

Examples: $\frac{3}{4}, \frac{-1}{9}, \frac{7}{5}, \frac{12}{7}$ etc

Irrational Numbers:

- Those numbers which cannot be represented in the form of p/q where p and q are integers and $q \neq 0$ are known as irrational numbers
- Examples: 1) 0.12123123412345123456.....
2) 5.67678678967890678901.....

Problem 1:

Find the value of $\sqrt{2}$ upto 3 decimal

1	2.000000	1	.	4	1	4
	1 ↓					
2	4	100				
		96 ↓				
28	1	400				
		281 ↓				
282	4	19900				
		11296				

$$\sqrt{2} = 1.414 \dots \text{(approximately)}$$

$$\sqrt{2} = 1.41421356 \dots$$

$\sqrt{2}$ is an irrational number

Home work:

- Find $\sqrt{3}, \sqrt{5}, \sqrt{7}$ approximate values upto 3 decimal places
 - Every Rational number can be expressed in to two forms
 - 1) Fraction form
 - 2) Decimal form

Example: $\frac{3}{4} = 0.75$

$\frac{3}{4}$ is fraction form and 0.75 is decimal form

Example: $\frac{5}{4} = 1.25$

Problems:

1. Convert into p/q form

$$3.75 = \frac{375}{100} = \frac{75}{20} = \frac{15}{4}$$

$$0.15 = \frac{15}{100} = \frac{3}{20}$$

2. Convert into decimal form

$$\frac{5}{32} = 0.15625 \qquad \sqrt[32]{5}$$

$$\frac{162}{5} = 32.4 \qquad \sqrt[32]{162}$$

Decimal Number

- Terminated decimal
- Non Terminated decimal

Terminated decimal:

0.125

3.75 and

5.62 are terminate decimals

Their p/q forms is/are

$$0.125 = \frac{1}{8} \longrightarrow \frac{125}{1000} = \frac{1}{8}$$

$$3.75 = \frac{15}{4} \longrightarrow \frac{375}{100} = \frac{15}{4}$$

$$5.62 = \frac{281}{50} \longrightarrow \frac{562}{100} = \frac{281}{50}$$

Non terminal decimal:

0.333333.....

1.345345345.....

3.67678978967890678901678902.....

These are non terminating decimals

Non Terminated decimal:

- Recurring decimal
- Non recurring decimal

Recurring decimal:

		Period	Period city
1.	0.333333..... ($0.\bar{3}$)	3	1
2.	2.555555..... ($2.\bar{5}$)	5	1

- | | | | | |
|----|------------------|-----------------------|------|---|
| 3. | 5.123123123..... | $(5.\overline{123})$ | 123 | 3 |
| 4. | 6.898989..... | $(6.\overline{89})$ | 89 | 2 |
| 5. | 8.56785678..... | $(8.\overline{5678})$ | 5678 | 4 |

These are Recurring decimals

Non Recurring decimals:

Examples:

- 5.01001000100001000001.....
- 0.12123123412345123456.....
- 23.58634213467912357.....

These are Non-Recurring decimals

Irrational Number:

“A irrational number is Non-Recurring and Non-Terminated decimal”

Examples: $\sqrt{7}, \sqrt{59}, 5.0100001200000123.....$

PROBLEMS

- Express $4.\overline{7}$ in the form of a rational number

Let $x = 4.\overline{7}$

$x = 4.7777.....$

Both sides multiply with 10

period = 7

$10x = 47.7777.....$

periodicity = 1

$x = 4.7777.....$

⊖

$9x = 43.0000$

$x = \frac{43}{9}$

Home Work:

Express in p/q form

- | | | | |
|----|--------------------|----|----------------------|
| 1. | $10.\overline{3}$ | 2. | $5.\overline{7}$ |
| 3. | $12.\overline{35}$ | 4. | $142.42\overline{6}$ |
- Let $0.4\overline{7}$ in the form of p/q

Let $x = 0.4\overline{7}$

$x = 0.474747.....$

Both sides multiply with 100

period = 47

$100x = 47.4747.....$

periodicity = 2

$x = 0.4747.....$

⊖

$99x = 47$

$x = \frac{47}{99}$

Home work:

- Express in p/q form 1) $18.9\overline{71}$ 2) $13.21\overline{45}$
- Represent the sum $3.\overline{36} + 5.\overline{04} - 6.\overline{3}$ in p/q form

PROBLEMS

1. Prove that $\sqrt{2}$ is irrational

Let $\sqrt{2}$ be rational

$$\sqrt{2} = p/q \quad (\text{HCF of } p, q = 1)$$

$$p = \sqrt{2}q$$

$$p^2 = 2q^2$$

2 is a factor of P^2

2 is a factor of P

$$P = 2K$$

$$P^2 = 4K^2$$

$$2q^2 = 4K^2$$

$$q^2 = 2K^2$$

2 is a factor of q^2

2 is also factor of q

\therefore P and q are not co-prime

So $\sqrt{2}$ cannot be rational

Hence $\sqrt{2}$ is irrational

Home work

1. Prove that $\sqrt{3}$ is an irrational
2. Prove that $\sqrt{5}$ is an irrational
2. Prove that $2 + \sqrt{3}$ is an irrational

We can prove $2 + \sqrt{3}$ is an irrational number by contradictory method.

Assume that $2 + \sqrt{3}$ is a rational numbers

Let $2 + \sqrt{3} = p/q$ where p, q are integers and $q \neq 0$ (HCF of p, q = 1)

$$\sqrt{3} = \frac{p}{q} - 2$$

$$\sqrt{3} = \frac{p-2q}{q}$$

Here LHS is an irrational and RHS is an rational number

$\left(\frac{p-2q}{q}\right)$ is also rational number

LHS \neq RHS

So our assumption $2 + \sqrt{3}$ is a rational number is incorrect

$\therefore 2 + \sqrt{3}$ is an irrational number

Home Work

1. Prove that $3 + \sqrt{5}$ is an irrational
2. Prove that $5 - \sqrt{7}$ is an irrational