Number system, Indices, Rationalising Surds:

1. If $\frac{1}{2+\sqrt{3}} = a - \sqrt{b}$, then find a and b?

Rationalise with Rationalising Factor $2-\sqrt{3}$

$$\frac{1}{2+\sqrt{3}} \times \frac{2-\sqrt{3}}{2-\sqrt{3}} = a - \sqrt{b}$$

$$\frac{2-\sqrt{3}}{(2)^2 - \left(\sqrt{3}\right)^2} = a - \sqrt{b}$$

$$\frac{2-\sqrt{3}}{4-3} = a - \sqrt{b}$$

$$2 - \sqrt{3} = a - \sqrt{b}$$

$$\therefore \ a = 2 \text{ and } b = 3$$

2. If $p = \frac{2+\sqrt{3}}{\sqrt{3}+1}$ and $q = \frac{2-\sqrt{3}}{\sqrt{3}-1}$, then find $p+q$

$$\frac{2+\sqrt{3}}{\sqrt{3}+1} + \frac{2-\sqrt{3}}{\sqrt{3}-1} = \frac{\left(2+\sqrt{3}\right)\left(\sqrt{3}-1\right) + \left(2-\sqrt{3}\right)\left(\sqrt{3}+1\right)}{\left(\sqrt{3}+1\right)\left(\sqrt{3}-1\right)}$$

$$= \frac{2\sqrt{3} - 2 + 3 - \sqrt{3} + 2\sqrt{3} + 2 - 3 - \sqrt{3}}{\left(\sqrt{3}\right)^2 - (1)^2}$$

$$= \frac{2\sqrt{3} - 0}{2} = \sqrt{3}$$

3. If $\frac{\sqrt{5}-1}{\sqrt{5}+1} + \frac{\sqrt{5}+1}{\sqrt{5}-1} = x + y\sqrt{5}$, then find x, y.

$$\frac{\left(\sqrt{5}-1\right)^2 + \left(\sqrt{5}+1\right)^2}{\left(\sqrt{5}+1\right)\left(\sqrt{5}-1\right)} = x + y\sqrt{5}$$

$$\frac{2\left(\left(\sqrt{5}\right)^2 + (1)^2\right)}{5-1} = x + y\sqrt{5}$$

$$\frac{2 \times 6}{4} = x + y\sqrt{5}$$

$$3 = x + y\sqrt{5}$$

$$3 + 0\sqrt{5} = x + y\sqrt{5}$$

$$\therefore \ x = 3 \text{ and } y = 0$$

1. If $x = \frac{1}{2-\sqrt{3}}$, then find the value of $x^3 - 2x^2 - 7x + 5$

2. If $\frac{7+3\sqrt{5}}{7-3\sqrt{5}} = a + b\sqrt{5}$, then find a and b

3. If $x = \frac{1}{7+4\sqrt{3}}$, $y = \frac{1}{7-4\sqrt{3}}$, then find $\frac{1}{x+1} + \frac{1}{y+1}$

4. If $a = \frac{2-\sqrt{5}}{2+\sqrt{5}}$, $b = \frac{2+\sqrt{5}}{2-\sqrt{5}}$, then find $a^2 - b^2$

5. If $a = \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}$, $b = \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$, then find $a^3 + b^3$

6. If $x = 9 + 4\sqrt{5}$ and $xy = 1$, if $\frac{1}{x^2} + \frac{1}{y^2} = k$, then find k.

Componendo and Dividendo Rule:

If a, b, c and d are in proportion

$\frac{a}{b} = \frac{c}{d}$

Then $\frac{a+b}{a-b} = \frac{c+d}{c-d}$

1. If $x = \frac{\sqrt{a+2b} + \sqrt{a-2b}}{\sqrt{a+2b} - \sqrt{a-2b}}$, then find $bx^2 - ax + b$.

$\frac{x}{1} = \frac{\sqrt{a+2b} + \sqrt{a-2b}}{\sqrt{a+2b} - \sqrt{a-2b}}$ apply componendo & dividend property

$\frac{x+1}{x-1} = \frac{\sqrt{a+2b} + \sqrt{a-2b} + \sqrt{a+2b} + \sqrt{a-2b}}{\sqrt{a+2b} - \sqrt{a-2b} - \sqrt{a+2b} + \sqrt{a-2b}}$

$\frac{x+1}{x-1} = \frac{2\sqrt{a+2b}}{2\sqrt{a+2b}}$ squaring on both sides

$\frac{(x+1)^2}{(x-1)^2} = \frac{a+2b}{a-2b}$ Again apply componendo & dividend

$\frac{(x+1)^2 + (x-1)^2}{(x+1)^2 - (x-1)^2} = \frac{a+2b+a-2b}{a+2b-a+2b}$

$\frac{2(x^2+1)}{4 \times x \times 1} = \frac{2a}{4b}$

$(x^2+1)b = ax$

$bx^2 - ax + b = 0$

HOME WORK

1. If $x = \frac{\sqrt{m+n} + \sqrt{m-n}}{\sqrt{m+n} - \sqrt{m-n}}$, then find $nx^2 - 2mx + n$

1. Find the value of $\frac{1}{1+\sqrt{2}}+\frac{1}{\sqrt{2}+\sqrt{3}}+\frac{1}{\sqrt{3}+\sqrt{4}}$

$\left(\frac{1}{1+\sqrt{2}}\times\frac{1-\sqrt{2}}{1-\sqrt{2}}\right)+\left(\frac{1}{\sqrt{2}+\sqrt{3}}\times\frac{\sqrt{2}-\sqrt{3}}{\sqrt{2}-\sqrt{3}}\right)+\left(\frac{1}{\sqrt{3}+\sqrt{4}}\times\frac{\sqrt{3}-\sqrt{4}}{\sqrt{3}-\sqrt{4}}\right)$

$\left(\frac{1-\sqrt{2}}{1-2}\right)+\left(\frac{\sqrt{2}-\sqrt{3}}{2-3}\right)+\left(\frac{\sqrt{3}-\sqrt{4}}{3-4}\right)$

$-1+\sqrt{2}-\sqrt{2}+\sqrt{3}-\sqrt{3}+\sqrt{4}$

$-1+2=1$

1. Find the value of

$\frac{1}{1+\sqrt{2}}+\frac{1}{\sqrt{2}+\sqrt{3}}+\frac{1}{\sqrt{3}+\sqrt{4}}+\frac{1}{\sqrt{5}+\sqrt{6}}+\frac{1}{\sqrt{6}+\sqrt{7}}+\frac{1}{\sqrt{7}+\sqrt{8}}+\frac{1}{\sqrt{8}+\sqrt{9}}$

2. Find the value of

$\frac{1}{2+\sqrt{3}}+\frac{2}{\sqrt{5}-\sqrt{3}}+\frac{1}{2-\sqrt{5}}$

3. If $x=\sqrt[3]{2+\sqrt{3}}$, then find $x^3+\frac{1}{x^3}$

4. Evaluate: $\frac{15}{\sqrt{10}+\sqrt{20}+\sqrt{40}-\sqrt{5}-\sqrt{80}}$

If $\sqrt{5}=2.236$ and $\sqrt{10}=3.162$

5. Evaluate: $\frac{1}{\sqrt[1]{2}+2\sqrt{1}}+\frac{1}{2\sqrt{3}+3\sqrt{2}}+\frac{1}{\sqrt[3]{4}+4\sqrt{3}}+.....+\frac{1}{399\sqrt{400}+400\sqrt{399}}$

1. Find the greater among $\sqrt{17}-\sqrt{12}$ and $\sqrt{11}-\sqrt{6}$

Rationalise with Rationalising factor

$\left(\sqrt{17}-\sqrt{12}\right)\times\frac{\left(\sqrt{17}+\sqrt{12}\right)}{\left(\sqrt{17}+\sqrt{12}\right)}, \qquad \left(\sqrt{11}-\sqrt{6}\right)\times\frac{\left(\sqrt{11}+\sqrt{6}\right)}{\left(\sqrt{11}+\sqrt{6}\right)}$

$\frac{\left(\sqrt{17}\right)^2-\left(\sqrt{12}\right)^2}{\sqrt{17}+\sqrt{12}}, \frac{\left(\sqrt{11}\right)^2-\left(\sqrt{6}\right)^2}{\sqrt{11}+\sqrt{6}}$

$\frac{17-12}{\sqrt{17}+\sqrt{12}}, \frac{11-6}{\sqrt{11}+\sqrt{6}}$

$\frac{5}{\sqrt{17}+\sqrt{12}}, \frac{5}{\sqrt{11}+\sqrt{6}}$ $\qquad \because \left(\sqrt{17}+\sqrt{12}\right)>\left(\sqrt{11}+\sqrt{6}\right)$

$\frac{5}{\sqrt{11}+\sqrt{6}}>\frac{5}{\sqrt{17}+\sqrt{12}}$

$\therefore \sqrt{11}-\sqrt{6}>\sqrt{17}-\sqrt{12}$

HW

1. Find the greater among $\sqrt{11}-\sqrt{5}$ and $\sqrt{19}-\sqrt{13}$

2. Find the square root of $7+4\sqrt{3}$ (or) $\sqrt{7+4\sqrt{3}}=?$

$$\sqrt{7+4\sqrt{3}}=\sqrt{7+2\times2\times\sqrt{3}}$$
$$=\sqrt{7+2\times\sqrt{4}\times\sqrt{3}}$$
$$=\sqrt{7+2\sqrt{12}}$$
$$=\sqrt{4+3+2\sqrt{4}\times\sqrt{3}}$$
$$=\sqrt{\left(\sqrt{4}\right)^2+\left(\sqrt{3}\right)^2+2\sqrt{4}\times\sqrt{3}}$$
$$=\sqrt{\left[\left(\sqrt{4}\right)+\left(\sqrt{3}\right)\right]^2}$$
$$=\sqrt{4}+\sqrt{3}$$
$$=2+\sqrt{3}$$

$$\sqrt{13-2\sqrt{30}}=\sqrt{10+3-2\sqrt{10}\sqrt{3}}$$
$$=\sqrt{\left(\sqrt{10}\right)^2+\left(\sqrt{3}\right)^2-2\sqrt{10}\sqrt{3}}$$
$$=\sqrt{10}-\sqrt{3}$$

HOME WORK

1. If $\left(a+b\sqrt{3}\right)^2=7+4\sqrt{3}$, find a and b

2. Evaluate $\sqrt{19-8\sqrt{3}}$

3. Find the square root of $7+\frac{5}{2}\sqrt{3}$

4. Find $\sqrt{3-2\sqrt{2}}$

5. If $\sqrt{13-x\sqrt{10}}=\sqrt{8}+\sqrt{5}$, then find x.