FIITJEE

KKP / MYP CENTRE

OLYMPIAD FOUNDATION
PRMO

Time: 3 hrs **Max. Marks : 100**

The goal of this paper is acquaint you with problem solving at Pre RMO level, infact at IITJEE level also. These questions will encourage you to learn how to approach problems, strategies to solve problems and to strengthen your problem solving skills.

You might not get answers to all the problems in first attempt. Keep trying again and again. We will upload key and solutions in the next two days for self evaluation.

This paper contains 30 questions and the answer for all these questions is an integer between 0 and 99. The first 25 questions carries 3 marks each and last 5 questions carries 5 marks each.

MATHEMATICS

1. Consider two successive even numbers. If 10 is subtracted from 4 times of the smaller one, then the resulting value is less than or equal to 2 times of the larger one. What is the maximum possible value of the sum of the numbers is?

2. Let $x_n, n \in \mathbb{N}$ be a positive integer. If $x_n + x_{n+1} = 100 \ \forall n \in \mathbb{N}$ and $x_{10} = 10$ then $x_{50} = ?$

3. The number of ordered pairs of natural numbers (x, y) such that $x + 3y = 100$.

4. How many non-negative integer values of `x' satisfy the equation $\left[\frac{x}{5}\right] = \left[\frac{x}{7}\right]$, where $[x]$ denotes greatest integer $\leq x$.

5. The real numbers a, b, c, d satisfy the equation $a+1 = b+2 = c+3 = d+4 = a+b+c+d-8$. Then the value of $a^2 + b^2 + c^2 + d^2$ is?

6. Consider the system of linear equations $x + y = 12 \quad y + z = 15 \quad z + x = 11$ then the value of $2x + 3y + 4z$ is

7. The sum of all the values of x which satisfy $\left(x^2 - 5x + 7\right)^{x^2 - 8x + 15} = 1$ are

8. If $ab \neq 0$ and $a^2 + 4b^2 = 5ab$ then the value of $\left|\frac{a+2b}{a-2b}\right|$ is

9. Let set $A = \{n \in \mathbb{N} / \ \frac{(n+1)(n+2)}{n-3}$ is a natural number $\}$ then $n(A)$ is

10. Let set $A = \{n \in \mathbb{N} / 1 \leq n \leq 2020,\ n^n$ is a perfect square$\}$ then sum of digits of $n(A)$ is

11. If $x = 70$ satisfies the equation $2|x-25| = x + a$ where a is constant. If $x = b \neq 70$ also satisfies the equation then $a + b$ is

12. If $|x| + x + y = 8$ and $x + |y| - y = 14$ then the value of $x + y$ is

13. The sum of the squares of the roots of the equation $x^2 - 5[x] + 3 = 0$.

14. To each element of set $S = \{1,2,3,.....,100\}$ .a colour is assigned, two elements a, b in S are of same colour if and only if $a - b$ is divisible by 5. Then the number of distinct colours required is

15. If $\frac{a}{b+c} + \frac{b}{c+a} + \frac{c}{a+b} = 1$, then the value of $\frac{a^2}{b+c} + \frac{b^2}{c+a} + \frac{c^2}{a+b}$ is

16. If $|a-1| < 2, |b-3| < 4, |c-5| < 6$ and $x < a+b+c < y$ then $x + y$ is

17. If $20 \leq a \leq 21$ then the value of $\left|2 - a - \left|6 - |a-20|\right|\right|$

18. If 'n' is the smallest positive integer such that $0.33 < \frac{m}{n} < \frac{1}{3}$ then the value of $n - m$ is

19. If $x = 3a + 2 = 4b + 3 = 5c + 4$, where a, b, c are all positive integers. Then the smallest possible value of x is

20. If $\left(1 - \frac{1}{2^2}\right)\left(1 - \frac{1}{3^2}\right)\left(1 - \frac{1}{4^2}\right).....\left(1 - \frac{1}{99^2}\right) = \frac{p}{q}$ then $q - p$ is

21. Given $|a-b| = 2$, $|b-c| = 3$, $|c-d| = 4$ where $a, b, c, d \in \mathbb{R}$. Then the sum of all possible values of $|a-d|$ is

22. The minimum value of the expression $|x-1| + |x-2| + |x-3| + |x-4|$ is

23. Suppose x and y are real numbers such that $|x| \geq |y|$, $|x-y| + 2x + 3y = 16$ and $|y-x| + 3x + 4y = 5$ then the value of $|x - 2y|$ is

24. Let $S=1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+.....-\frac{1}{2020}+\frac{1}{2021}$, $P=\frac{1}{1011}+\frac{1}{1012}+\frac{1}{1013}+.....+\frac{1}{2021}$

Then the value of $(S-P)^{2022}$ is

25. How many natural numbers 'n' which are less than 2020 for which n is divisible by 5, $n+1$ is divisible by 6, $n+2$ is divisible by 7

26. If $\sum_{n=1}^{6}|x-2n|=18$ for all $x\in[a,b]$ then $a+b$ is

27. If $x=\Big|...\Big|\big|99-1\big|-2\Big|-3\Big|...-98\Big|-99\Big|$ then the value of x is

28. x,y,z are positive real numbers such that $[x]-y=2[y]-z=3[z]-x=\frac{2}{3}$ then the value of $x+y+z$ is

29. If $\frac{2^2}{2^2-1}.\frac{3^2}{3^2-1}.\frac{4^2}{4^2-1}.....\frac{2020^2}{2020^2-1}=\frac{p}{q}$ where p,q are relatively prime then the sum of the digits in $p+q$ is

30. The number of natural numbers n such that $\sqrt{n}+\sqrt{n+200}$ is a natural number

✻ *Wish You* est *all the Best* ✻