NUMBER SYSTEM

The Set of Natural Numbers

Numbers, which are used in counting are called natural numbers or positive integers. The set of natural numbers are denoted by $\mathbb{N}$.

$$\therefore \mathbb{N} = \{1, 2, 3, 4, ...\}$$

The Set of Whole Numbers

The set of natural numbers along with number '0' (zero) is called whole numbers. The set of whole numbers are denoted by W.

$$\therefore W = \{0, 1, 2, 3, ...\}$$

The Set of Integers

The set of natural numbers, their negatives and along with zero are considered as integers denoted by $\mathbb{Z}$.

$$\therefore \mathbb{Z} = \{....., -3, -2, -1, 0, 1, 2, 3, ...\}$$

The Set of Rational Numbers

The numbers of the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$ form the set of rational numbers denoted by $\mathbb{Q}$

$$\therefore \mathbb{Q} = \left\{ \frac{p}{q} / p, q \in \mathbb{Z}, q \neq 0 \right\}$$

Examples: $\frac{2}{5}, -\frac{5}{9}, \frac{17}{-6}, \frac{0}{3}$ etc.

Note: The rational numbers are either terminating or recurring decimals

Ex: $\frac{21}{4} = 5.25$ is terminating decimal

$\frac{40}{3} = 13.333......,$ recurring decimal

The Set of Irrational Numbers

There are numbers, which cannot be expressed in the form $\frac{p}{q}$, where p, q are integers. They are called irrational numbers and are denoted by Q' or Q^c

Examples:

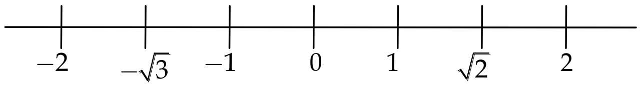
1. 2.323323332333……..
2. 1234.5678910111213……
3. $\sqrt{2}, \sqrt{3}, e, \pi$ etc

The Set of Real Numbers

The set of rational numbers together with the set of irrational numbers is called the set of real numbers denoted by $\mathbb{R}$.

Real Number line

A real number line is an ordinary geometric straight line whose points have been identified with set $\mathbb{R}$ of real number.



Here to every real number there corresponds a point on the line and conversely, for every point on the line there corresponds a real number.

Even And Odd Number

Every integer which is exactly divisible by '2' is called even number otherwise it is called odd number.

Every even number is of the form $2n$, n is an integer and odd number is of the form $2n+1$, n is an integer.

Prime And Composite Numbers

A natural number which has exactly two factors (1 and itself) is called prime number. A natural number which has three or more factors is called a composite number.

Ex: 1. $2,3,5,7,11,13,.......$ are prime numbers

2. $4,6,8,9,12,15,.....$ are composite numbers

3. 1 is neither prime nor composite

Coprime Numbers

Two natural numbers are said to be coprime if they don't have any common factor expect 1.

Ex: 8, 15 are coprime

Intervals

If we wish to consider all the real numbers between a and b (with or without including one or both the points a and b) then such sets are called intervals.

Open interval (*a*, *b*): If a and b are real numbers with $a<b$, we denote by (a,b) the set of all real numbers between a and b excluding a and b

$$\therefore (a,b)=\{x / x\in\mathbb{R}, a<x<b\}$$

Closed interval [*a*, *b*]: If a and b are real numbers with $a<b$, we denote by $[a,b]$ the set of all real numbers between a and b including a and b

$$\therefore [a,b]=\{x / x\in\mathbb{R},\ a\le x\le b\}$$

Half open intervals: The interval $(a,b]$, which is open at $'a'$ and closed at $'b'$ is called half open interval and denotes all real numbers such that $a<x\le b$ similarly $[a,b)$ denotes all real numbers x such that $a\le x<b$

Other intervals are

$$(a,\infty)=\{x / x\in\mathbb{R}, x>a\}$$

$$(-\infty,b)=\{x / x\in\mathbb{R}, x<b\}$$

$$[a,\infty)=\{x / x\in\mathbb{R}, x\ge a\}$$

$$(-\infty,b]=\{x / x\in\mathbb{R}, x\le b$$

The set of all real numbers can be denoted by $(-\infty,\infty)$

Inequality

If $a<b$ then, $ac<bc$ if $c>0$, $ac>bc$ if $c<0$, $ac=bc$ if c = 0

Practice Problems

1. Find the value of x satisfying the equation $-3x-7\le 5$

2. If $-2<x\le 3$ then find the interval in which $2-3x$ lies.

3. If $-3\le 5-4x<21$ then find the interval in which x lies.

4. If $\frac{2x-3}{4}+1>2+\frac{1+4x}{3}$ then find the interval in which x lies.

5. The values of x satisfying both the equations $x+5\ge 2(x+1)\, and\, 2-x<3(x+2)$

6. If $x<-x<4x+10$ then find the interval in which x lies.

7. If $16<x^2<25$ then find the interval in which x lies.

8. If $-x\le x\le -2x$ then find the interval in which x lies.

9. If $x\in(-2,5)$ then find the interval in which x^2 lies

10. If $x\in[-6,3]$ then find the interval in which x^2 lies.

11. If $x\in[1,4]$ then find the interval in which $\frac{1}{x}$ lies

12. if $x\in[-3,4]$ then find the interval in which $\frac{1}{x}$ lies.

KEY

1.	$[-4,\infty)$	2.	$[-7,8)$	3.	$(-4,2]$
4.	$\left(-\infty,-\frac{5}{2}\right)$	5.	$(-1,3]$	6.	$(-2,0)$
7.	$(-5,-4)\cup(4,5)$	8.	0	9.	$[0,25]$
10.	$[0,36]$	11.	$\left[\frac{1}{4},1\right]$	12.	$\left(-\infty,-\frac{1}{3}\right)\cup\left(\frac{1}{4},\infty\right)$

OBJECTIVE QUESTIONS

1. Let $ABCDEFGHIJ$ be a 10 digit number where all digits are distinct. Further $A>B>C$, $A+B+C=9$, $D>E>F>G$ are odd numbers and $H>I>J$ are even numbers. Then which of the following is definitely wrong

(A) $E=5$ (B) $G=3$ (C) $H=6$ (D) $I=2$

2. When the product of four consecutive odd positive integers divided by 4, the set of remainders are

(A) $\{0\}$ (B) $\{1\}$ (C) $\{0,1\}$ (D) None of these

3. The number of solutions of the equations $a^2+1\geq 2b$, $b^2+1\geq 2a$ are

(A) 0 (B) 1 (C) 2 (D) Infinite

4. Let $P>3$ be a prime number. Then which of the following is always FALSE

(A) $P+2$ is a prime number

(B) $P+4$ is a prime number

(C) both $P+2$ and $P+4$ are prime numbers

(D) neither $P+2$ nor $P+4$ are prime numbers

5. Suppose $'x'$ is an irrational number and a,b,c,d are rational numbers. If $\frac{ax+b}{cx+d}$ is a rational then we must have

(A) $a=c=0$ (B) $a=c,b=d$ (C) $ad=bc$ (D) $a+d=b+c$

6. How many possible triplets (a,b,c) such that $ab=c,\ bc=a,\ ca=b$ and $a,b,c\in\mathbb{R}$ are

(A) 3 (B) 5 (C) 8 (D) 9

7. If a,b,c,d be real numbers such the $b\neq 0, d\neq 0$, $\frac{a}{b}<\frac{c}{d}$. Then which of the following is always TRUE

(A) $\frac{a}{b}<\frac{a-c}{b-d}<\frac{c}{d}$ (B) $\frac{a}{b}<\frac{a+c}{b+d}<\frac{c}{d}$ (C) $\frac{a}{b}<\frac{a-c}{b+d}<\frac{c}{d}$ (D) $\frac{a}{b}<\frac{a+c}{b-d}<\frac{c}{d}$

8. Consider the statements

$x(\alpha-x)<y(\alpha-y)$, $0<x<y<1$, for all x,y. Then which of the following is TRUE

(A) if and only if $\alpha\geq 2$ (B) if and only if $\alpha>2$

(C) if and only if $\alpha\leq 2$ (D) if and only if $\alpha<2$

9. If $\frac{a+b}{b+c}=\frac{c+d}{d+a}$ then which of the following is true where $a\neq c$

(A) $a+c=b+d$ (B) $ac=bd$ (C) $a+b+c+d=0$ (D) None of these

10. Let p,q and s be integers such that $p^2=sq^2$ then

(A) p is even number (B) s divides p

(C) q^2 divides p (D) If s divides p then s is perfect square

KEY

1.	C	2.	B	3.	D	4.	C
5.	C	6.	B	7.	B	8.	A
9.	C	10.	D				

SET THEORY

Definition of Set

A well defined collection of objects is called a *set*.

Examples

1. The roots of the equation $x^2-5x+6=0$
2. The English alphabets from a to z
3. The set of natural numbers

In all these collections, we can identify each object precisely and hence they represent sets.

On the other hand, *honest people*, *clever students*, *handsome boys* and so on are relative terms and it is not possible to identify a particular person belongs to the *set* or not. Hence, they do not form sets in the language of mathematics.

Elements of a Set

The objects that belong to a *set* are called elements or members of the *set*.

Set notations

The sets are usually denoted by capital letters $A,B,C,......$ and their elements by $a,b,c,.......$

If a particular element x belong to set A. Then we write it as $x\in A$, if x and y belongs to set A, we write it as $x,y\in A$. However, if an element x does not belong to B, we write $x\notin B$.

We use *curly brackets* to enclose the elements of a set. For example

$C=${set of all even natural numbers} $=\{2,4,6,8,.......\}$

= { x/x is even natural number]

The symbol / stands for 'such that'

Each element in a *set* is separated by comma

Specifying sets

(i) Roster method or Listing method

In this method, a *set* is represented by listing all the elements within $\{\ \}$. For example $A=\{a,e,i,o,u\}$

(ii) Set builder method or Rule method

In this method, we state one or more properties of the elements so that we can decide whether an object belongs or not belongs to the *set*. For example

$B=\{x\,|\,x$ is single digit natural number}

Singleton set

The set which contains only one element is called *singleton set*. For example

$D=\{x/x$ is root of $x^2-4x+4=0\}$

Empty Set or Null Set

The *Set* which contains no element is called the *empty set*. The null set is denoted by ϕ or $\{\ \}$ for example $E=$ { x/x is real root of $x^2+1=0\}$ is the *null set*.

The cardinal number of a set

If a set A contain finite number of elements n, we denote the cardinal number of set by $n(A)$.

In other words, $n(A)$ stands for number of elements in a finite set A.

For example $A=\{a,e,i,o,u\}$ then $n(A)=5$

$B=$ {x / x is single digit natural number} then $n(B)=9$

The cardinal number of the *empty set* is *zero*.

Subset of a set

Two sets A and B are such that, each element of set A is also, an element of set B then A is called a subset of B, denoted by $A \subseteq B$ (read it as A is subset of B)

Thus a set A is a subset of B if $x \in A \Rightarrow x \in B$

If A is not a subset of B, we write it as $A \not\subseteq B$

Note: For every set, the empty set and the set itself are subsets.

Example:

For $A=\{1,2,3\}$, the sets $\{\ \}$, $\{1\},\{2\},\{3\},\{1,2\},\{2,3\},\{1,3\}$, $\{1,2,3\}$ are all subsets.

Equality of sets

Two sets A and B are said to be equal if every element of set A is an element of set B and every element of set B is an element of set A

i.e $x \in A \Leftrightarrow x \in B$

Note: From the above definition we can conclude the following

1. A set does not change if we change the order in which the elements are tabulated
2. A set does not change if one or more elements are repeated.

For example consider

$A=\{1,2,3,4\}, B=\{3,1,4,2,1,3\}$ then by definition $A=B$

Proper Subset

Consider the set A which is subset of set B. If there is atleast one element of B which is not in the set A, then A is called proper subset of B and it is denoted by $A \subset B$

Set of Sets

A *set* whose elements are *set(s)* is called set of *sets*.

Example:

$\{\phi,\{1\},\{2\},\{2,3\}\}$ is set of sets.

Power Set

Let S is any *set*. Then the family of the all subsets of S is called the *power set* of S and is denoted by $P(S)$

Let $S=\{1,2,3\}$, then $P(S)=\{\{\ \},\{1\},\{2\},\{3\},\{1,2\},\{1,3\},\{2,3\},\{1,2,3\}\}$

Note: If a set S has $'n'$ elements then it power set $P(S)$ has 2^n elements

Universal Set

In any discussion about *sets*, all the *sets* under consideration are *subsets* of a particular *set*. Such *set* is called *universal set* denoted by U.

The Union of two sets A and B

It is the *set* consisting of precisely those elements which are belongs to either A or B or both and is denoted by $A \cup B$

$\therefore A \cup B = \{x / x \in A \text{ or } B \text{ or both}\}$

Example:

If $A = \{1,2,3\}$, $B = \{3,4,5\}$ then $A \cup B = \{1,2,3,4,5\}$

The Intersection of two sets A and B

It is the *set* of all those elements that belong to both A and B and denoted by $A \cap B$

$\therefore A \cap B = \{x / x \in A \text{ and } B \text{ both}\}$

Example:

$A = \{1,2,3,4\}, B = \{2,4,6,8\}$ then $A \cap B = \{2,4\}$

Disjoint Sets

Two sets A and B are said to be disjoint if they don't have any element in common. In other words $A \cap B = \phi$

Difference of two Sets A and B

The difference of two sets A and B in that order is the set of elements that belong to A but that not belong to B and is denoted by $A - B$.

$\therefore A - B = \{x \mid x \in A \text{ } and \text{ } x \notin B\}$

Similarly $B - A = \{x | x \in B \text{ } and \text{ } x \notin A|\}$

Example:

$A = \{1,2,3,4\}, B]\{2,4,6,8\}$ then $A - B = \{1,3\}, B - A = \{6,8\}$

In general $A - B \neq B - A$

Complement of Set

The complement of set A is denoted by A' or A^c or $\overline{A}$ and is defined as the difference universal set U and the set A.

Example:

$\mathrm{U} = N = \{1,2,3,4,......\}, A = \{2,4,6,8,.....\}$ then $A' = \mathrm{U} - A = \{1,3,5,7,.......\}$

Symmetric Difference of Sets

The symmetric difference of two sets A and B is denoted by $A \Delta B$ and is defined as $(A - B) \cup (B - A)$.

$\therefore A \Delta B = (A - B) \cup (B - A) \ = (A \cup B) - (A \cap B)$

$A \Delta B = \{x / x \in A \ x \notin B \text{ or } x \notin A \text{ and } x \in B\}$

Example: $A = \{1,2,3,4\}$, $B = \{2,4,6,8\}$, then $A \Delta B = \{1,3,6,8\}$

Algebra of Sets: Let A, B and C be any three sets and 'U' be the universal set. Then

Idempotent Laws: a) $A \cup A = A$, b) $A \cap A = A$

Identity Laws: a) $A \cup \phi = A$, b) $A \cap \mathrm{U} = A$

Commutative Laws: a) $A \cup B = B \cup A$, b) $A \cap B = B \cap A$

Associative Laws: a) $(A \cup B) \cup C = A \cup (B \cup C)$, b) $(A \cap B) \cap C = A \cap (B \cap C)$

Distributive Laws: a) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$, b) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

De-Morgan Laws: a) $(A \cup B)' = A' \cap B'$, b) $(A \cap B)' = A' \cup B'$

Useful Results:

(i) $A - B = A \cap B'$ (ii) $B - A = B \cap A'$ (iii) $A - B = A \Leftrightarrow A \cap B = \phi$

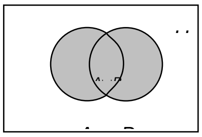
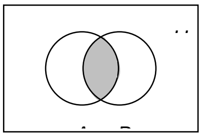
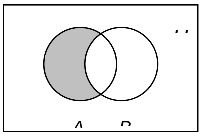
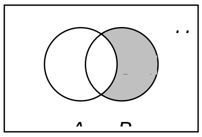
(iv) $(A - B) \cup B = A \cup B$ (v) $(A - B) \cap B = \phi$ (vi) $A \subseteq B \Leftrightarrow B' \subseteq A'$

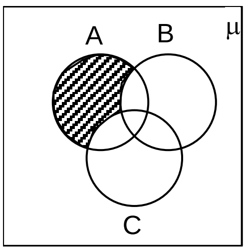
(vii) $A - (B \cup C) = (A - B) \cap (A - C)$ (viii) $A - (B \cap C) = (A - B) \cup (A - C)$

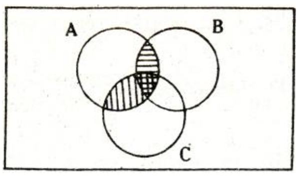
(ix) $A \cap (B - C) = (A \cap B) - (A \cap C)$ (x) $A \cap (B \cap C) = (A \cap B) \cap (A \cap C)$

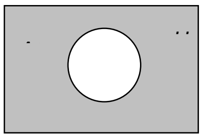
Venn Diagrams

Venn diagrams are used to represent sets by circles (or any closed shape) inside a rectangle. The rectangle represent the universal set and the circle represent the respective set mentioned. The element of the set are points inside the circle. The Venn diagrams for some sets are as follows.







$A - (B \cup C)$ $A \cap (B \cup C)$ $\mathrm{U} - A$

Some important results on number of elements in Sets:

If A, B and C are finite sets and U be the universal sets, then

i) $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

ii) $n(A \cup B) = n(A) + n(B) \Leftrightarrow A, B$ are disjoint sets

iii) $n(A-B)=n(A)-n(A\cap B)$

iv) $n(A\Delta B)=n(A)+n(B)-2n(A\cap B)$ = Number of elements which belongs to exactly one of A or B

v) $n(A\cup B\cup C)=n(A)+n(B)+n(C)-n(A\cap B)-n(B\cap C)-n(C\cap A)+n(A\cap B\cap C)$

vi) Number of elements in exactly two of the sets $A,B,C=n(A\cap B)+n(B\cap C)+n(C\cap A)-3n(A\cap B\cap C)$

vii) Number of elements is exactly one of the sets A,B,C

$$=n(A)+n(B)+n(C)-2n(A\cap B)-2n(B\cap C)-2n(C\cap A)+3n(A\cap B\cap C)$$

viii) $n(A'\cup B')=n((A\cap B)')=n(\mathrm{U})-n(A\cap B)$

ix) $n(A'\cap B')=n((A\cup B)')=n(\mathrm{U})-n(A\cup B)$

x) $n(A_1\cup A_2\cup......\cup A_n)$

$$=\sum n(A_i)-\sum n\left(A_i\cap A_j\right)+\sum n\left(A_i\cap A_j\cap A_k\right).......+(-1)^{n+1}n(A_1\cap A_2\cap....\cap A_n)$$

OBJECTIVE QUESTIONS

1. If A and B are two sets containing 3 and 6 elements respectively, then the minimum number of elements in $A\cup B$ is

(A) 3 (B) 4 (C) 6 (D) 12

2. If A and B are two sets containing 4 and 13 elements respectively, then the maximum number of elements in $A\cup B$ is

(A) 13 (B) 17 (C) 4 (D) 21

3. A survey shows that 63% of the Americans like football where as 76% like Basket ball. If $x\%$ like both foot ball and basket ball, then

(A) $x=63$ only (B) $x=39$ only (C) $x\in\{39,40,....,63\}$ (D) none

4. If $2N=\{2x:x\in N\}$ and $3N=\{3x:x\in N\}$ then $2N\cap 3N=$

(A) N (B) 6N (C) 2N (D) 3N

5. If $X=\left\{4^n-3n-1,n\in N\right\}$ and $Y=\left\{9(n-1),n\in N\right\}$ where N is the set of natural numbers, then $X\cup Y$ is

(A) N (B) X (C) Y (D) $Y-X$

6. If $a\mathbb{N}=\{ax:x\in\mathbb{N}\}$ and $b\mathbb{N}\cap c\mathbb{N}=d\mathbb{N}$, where $b,c\in\mathbb{N}$ are relatively prime, then

(A) $d=bc$ (B) $c=bd$ (C) $b=cd$ (D) none of these

7. Two finite sets have m and n elements. The total number of subsets of the first set is 56 more than the total number of subsets of second set. The values of m and n are

(A) $m=7,n=6$ (B) $m=6,n=3$ (C) $m=5,n=1$ (D) $m=8,n=7$

8. Which of the following is empty set?

(A) $\{x : x \in \mathbb{R} \text{ and } x^2 - 1 = 0\}$ (B) $\{x : x \in \mathbb{R} \text{ and } x^2 + 1 = 0\}$

(C) $\{x : x \in \mathbb{R} \text{ and } x^2 - 9 = 0\}$ (D) $\{x : x \in \mathbb{R} \text{ and } x^2 = x + 2\}$

9. Let sets X and Y are sets of all positive divisors of 400 and 1000 respectively. Then $n(X \cap Y) =$

(A) 6 (B) 8 (C) 10 (D) 12

10. If X and Y are two sets, then $X \cap (X \cup Y)'$ equals

(A) X (B) Y (C) ϕ (D) none of these

11. If $A = \{a, \{b\}\}$, then $P(A) =$

(A) $\{\{a\}, \{\{b\}\}\}$ (B) $\{\phi, \{a\}, \{b\}, \{a, b\}\}$ (C) $\{\phi, \{a\}, \{\{b\}\}, A\}$ (D) none of these

12. In a group of athletic teams in a school, 21 are in the basket ball team, 26 in the hockey team, and 29 in food ball team. If 10 play hockey and basket ball; 12 play foot ball and basket ball; 15 play hockey and foot ball and 8 play all the three games, then the total number of players are

(A) 43 (B) 45 (C) 47 (D) 49

13. The number of elements in the set $P\{P[P(\phi)]\}$ is

(A) 0 (B) 2 (C) 3 (D) 4

14. Let $S = \{x / x$ is an integer between 1 and 1000 (including both) which are neither perfect square nor perfect cube$\}$ Then the number of elements is S are

(A) 959 (B) 960 (C) 961 (D) 962

15. In a town of 10,000 families, it was found that 40% families buy news paper A, 20% families buy newspaper B and 10% families buy newspaper C, 5% buy A and B, 3% buy B and C, and 4% buy C and A. If 2% buy all the three, then the number of families which buy none of A, B and C is

(A) 4,000 (B) 3,300 (C) 4,200 (D) 5,000

16. The average of all the numbers in the set $S = \{|a - b| / a, b \in \{1, 2, 3, ...9\}\}$ is

(A) 4 (B) 8 (C) 45 (D) None

17. Suppose $A_1, A_2, ..., A_{30}$ are 30 sets each having 5 elements and $B_1, B_2, ..., B_n$ are n sets each with 3 elements. Let $\bigcup_{i=1}^{30} A_i = \bigcup_{j=1}^{n} B_j = S$ and each element of S belongs to exactly 10 of the A_i's and exactly 9 of the B_i's. Then n is equal to

(A) 15 (B) 30 (C) 45 (D) none of these

18. Let $S = \{1, 2, 3, 4\}$. The total number of unordered pairs of disjoint subsets of S is equal to

(A) 26 (B) 34 (C) 41 (D) 42

19. If A and B are two sets then $A \cap (A \cap B)^C$ is equal to

(A) ϕ (B) A (C) B (D) $A \cap B^C$

20. In a class of 80 students numbered 1 to 80; all odd numbered students opt for cricket, students whose numbers are divisible by 5 opt for football and these whose numbers are divisible by 7 opt for Hockey. The number of students who do not opt any of the three games.

(A) 13 (B) 24 (C) 28 (D) 52

KEY

1.	C	2.	B	3.	C	4.	B	5.	C
6.	A	7.	B	8.	B	9.	D	10.	C
11.	C	12.	C	13.	D	14.	D	15.	A
16.	A	17.	C	18.	C	19.	D	20.	C

Modulus or Absolute value of a real number

For any $x \in \mathbb{R}$, the modulus (absolute) of x denoted by $|x|$ defined as $|x| = \begin{cases} x, & \text{if } x \ge 0 \\ -x, & \text{if } x < 0 \end{cases}$

Ex-1: $|5| = 5$, $|-6| = 6$ $\quad |8.3| = 8.3, |-9.4| = 9.4$

On real number line the modulus or absolute value of $'x'$ denotes the distance from 0 (zero) to $'x'$. Similarly $|x_1 - x_2|$ denotes, the distance between the numbers x_1 and x_2 on the real number line.

Ex-2: Solve $|x+1| + |x-3| = 8$

Sol. Case (i) $x \ge 3$

Then equation becomes $x + 1 + x - 3 = 8 \Rightarrow x = 5$

Case (ii) $x \in (-1, 3)$

Then $|x+1| = x+1, |x-3| = -(x-3)$

Given equation becomes $x + 1 - x + 3 = 8$

$\Rightarrow 4 = 8$, a contradiction

$\therefore$ No value of $x \in (-1, 3)$ satisfies

Case (iii) $x \le -1$

Then $|x+1| = -(x+1), |x-3| = -(x-3)$

Given equation becomes $-x - 1 - x + 3 = 8$

$\Rightarrow x = -3$

$\therefore$ The solution set is $x \in \{-3, 5\}$

Ex-3. Solve $|x+1| + |x-3| = 4$

Sol. $|x+1|$ denotes the distance from –1 to x

$|x+3|$ denotes the distance from 3 to x

$\therefore$ $|x+1| + |x-3|$ denotes the sum of the distances from -1 and 3 to x

On the real number line for all the points between –1 and 3 (including both) the sum of the distance is 4 only

$\therefore x \in [-1, 3]$

Ex-4. Solve $|x+1|+|x-3|=2$

Sol. As above $x \in \phi$, because there is no point on real line such that the sum of distance from –1 and 3 to x is 2.

General Results on Modulus

1. For any real number x, $\sqrt{x^2}=|x|$

2. For $a>0$

   (i) $|x| \le a \Leftrightarrow -a \le x \le a$

   (ii) $|x| \ge a \Leftrightarrow x \in (-\infty,-a] \cup [a,\infty)$

   (iii) $a \le |x| \le b \Leftrightarrow x \in [-b,-a] \cup [a,b]$

3. For $a<0$

   (i) $|x| \le a \Leftrightarrow x \in \phi$ (ii) $|x| \ge a \Leftrightarrow x \in \mathbb{R}$

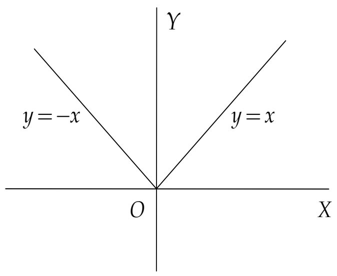
4. $|x+y|=|x|+|y| \Leftrightarrow xy \ge 0$

5. $|x-y|=|x|-|y| \Leftrightarrow xy \le 0$

6. (i) $|x \pm y| \le |x|+|y|$ (ii) $|x \pm y| \ge \left||x|-|y|\right|$

Graph of $|x|$

The graph of $y=|x|$ is as shown in the figure



Examples

Solve the following Problems

1. $|x|=2$
2. $|x|=-5$
3. $|x|<3$
4. $|x|>4$
5. $|x|<-6$
6. $|x|>-1$
7. $|x+3|=7$
8. $|x-2| \le 5$
9. $|x-1| \ge 3$
10. $|x-3|+|x-5|=12$
11. $|x-4|+|x+6|=10$
12. $|x+2|+|x+7|=4$
13. $|x-1|+|x+3|-|x-6|=12$
14. $|x-2|+3|x+1|=8$
15. $|x+1|+2|x+2|+3|x+3|=15$
16. $|x+1|+|x+4| \le 8$

17. $|x+2|+|x-4|\geq 10$

18. $|x+1|-|x+5|<2$

19. $|2x-1|=|x+3|$

20. $|x-1|+|x|+|x+1|=x+2$

21. $2\leq|x-4|\leq 5$

22. $-3\leq|x+2|\leq 4$

23. $||x-2|-1|\geq 3$

24. Prove that $|a+b|\leq|a|+|b|$ and equality holds only when $ab\geq 0$

25. Prove that $|a-b|\leq|a|+|b|$ and equality holds only when $ab\leq 0$

KEY

1.	± 2	2.	ϕ	3.	$(-3,3)$
4.	$(-\infty,-4)\cup(4,\infty)$	5.	ϕ	6.	$\mathbb{R}$
7.	$-10,4$	8.	$[-3,7]$	9.	$(-\infty,-2]\cup[4,\infty]$
10.	$-2,10$	11.	$[-6,4]$	12.	ϕ
13.	$\frac{22}{3}$	14.	$-\frac{9}{4},\frac{3}{2}$	15.	$-\frac{29}{6},\frac{1}{6}$
16.	$[-6.5,1.5]$	17.	$(-\infty,-4]\cup[6,\infty)$	18.	$(-4,\infty)$
19.	$4,-\frac{2}{3}$	20.	$[0,1]$	21.	$[-1,2]\cup[6,9]$
22.	$[-6,2]$	23.	$(-\infty,-2]\cup[6,\infty)$		

Greatest Integer Function or Step function

The greatest integer function of x is denoted by $[x]$ is defined as the greatest integer less than or equal to 'x'.

Examples:

1.	$[5.8]=5$	2.	$[-3.1]=-4$	3.	$[7]=7$
4.	$[-6]=-6$	5.	$[\sqrt{2}]=1$	6.	$[-\pi]=-4$

Properties of $[x]$

(i) $[x]\leq x<[x]+1$

(ii) $x-1<[x]\leq x$

(iii) $[x]+[-x]=\begin{cases} 0 \text{ if } x\in\mathbb{Z} \\ -1 \text{ if } x\notin\mathbb{Z}\end{cases}$

(iv) $[x]-[-x]=\begin{cases} 2x & \text{if } x\in\mathbb{Z} \\ 2x+1 & \text{if } x\notin\mathbb{Z} \end{cases}$

(v) $[x]=\left[\dfrac{x}{2}\right]+\left[\dfrac{x+1}{2}\right]$

(vi) $\left[\dfrac{n+1}{2}\right]+\left[\dfrac{n+2}{4}\right]+\left[\dfrac{n+4}{8}\right]+......=n,\ n\in\mathbb{N}$

(vii) $[x]+\left[x+\dfrac{1}{n}\right]+\left[x+\dfrac{2}{n}\right]+......+\left[x+\dfrac{n-1}{n}\right]=[nx],\quad n\in\mathbb{N}$

(viii) (a) $[x]>n\Leftrightarrow x\in[n+1,\infty),\ n\in\mathbb{Z}$

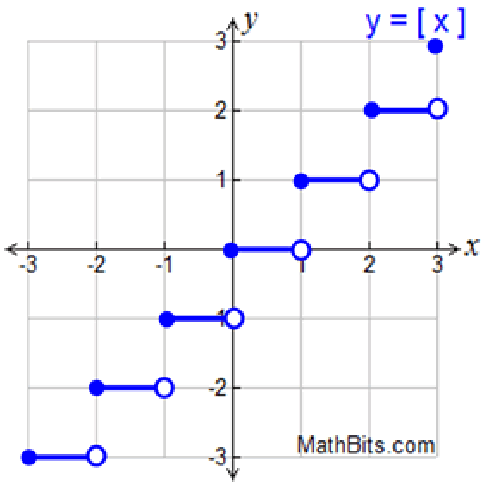
(b) $[x]\geq n\Leftrightarrow x\in[n,\infty),\ n\in\mathbb{Z}$

(ix) (a) $[x]<n\Leftrightarrow x\in(\infty,n),\ n\in\mathbb{Z}$

(b) $[x]\leq n\Leftrightarrow x\in(\infty,n+1),\ n\in\mathbb{Z}$

(x) $[x\pm n]=[x]\pm n\quad\forall\ \ n\in\mathbb{Z}$

Graph of $y=[x]$ is as shown in the figure



Fractional part function

The fractional part function of x is denoted by $\{x\}$ and defined as $\{x\}=x-[x]$

Ex: $\{2.3\}=0.3$

$\{-2.6\}=0.4$

Properties

(i) $\{x\}=x$, if $0\leq x<1$

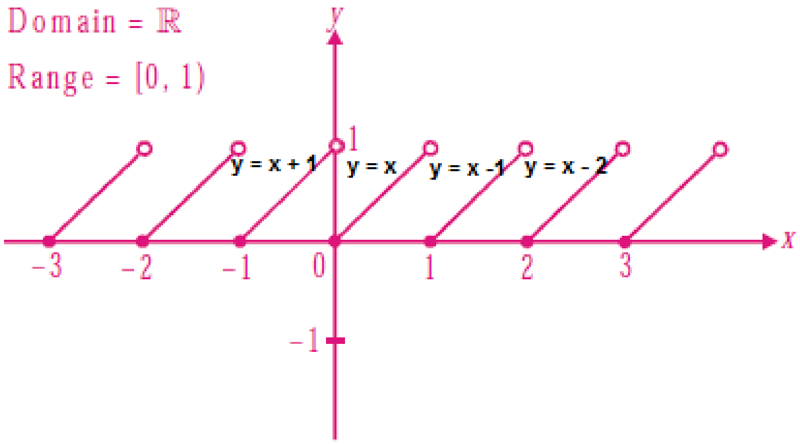
(ii) $\{x\}=0$, if $x\in\mathbb{Z}$

(iii) $\{-x\}=1-\{x\}$, if $x\notin\mathbb{Z}$

(iv) $\{x+n\}=\{x\},\ n\in\mathbb{Z}$

Graph of $\{x\}$

The graph of $\{x\}$ is as shown in the figure



Problems

Find the values of the following

1. $[5.7]+[-8.2]$
2. $[4.6]-[-6.4]$
3. $[\sqrt{2}]+[-\sqrt{2}]$
4. $[\pi]+[-\pi]$
5. $[3.4]+\{3.4\}$
6. $[2.7]-\{2.7\}$
7. $[1.9]+\{-1.9\}$
8. $[3.5]-\{-3.5\}$
9. $[-4.7]+\{-4.7\}$
10. $[-8.3]-\{-8.3\}$
11. $[x+2]+[x+3]=23$ then find x
12. $[x+4]+\{x+5\}=45$ then find x
13. If $2\le[x]\le 3$ then find x
14. Solve $4\{x\}=x+[x]$
15. Find the value of $[x]+\sum_{k=1}^{2020}\frac{\{x+k\}}{2020}$ where $[\ ],\{\ \}$ denotes the GIF, FPF respectively.

Answers

1.	-4	2.	11	3.	-1
4.	-1	5.	3.4	6.	2.4
7.	1.1	8.	3	9.	-4.7
10.	-9.7	11.	$[9,10)$	12.	41
13.	$[2,4)$	14.	$0,\frac{5}{3}$	15.	x

Wavy Curve Method

Let $y=\frac{f(x)}{g(x)}$ be an expression in $'x'$ where $f(x)$ and $g(x)$ are polynomials in $'x'$. Now, if it is given that $y>0$ or $y\geq 0$ or $y<0$ or $y\leq 0$ then we need to find all the values of $'x'$ which satisfies the given inequality. The solution can be found in the following steps.

Step I:

Factorise $f(x)$ and $g(x)$ and let y is of the below form

$$y=\frac{(x-a_1)^{m1}(x-a_2)^{m2}.......(x-a_p)^{m_p}}{(x-b_1)^{n1}(x-b_2)^{n2}.......(x-b_q)^{n_q}}$$

where $m_1, m_2, ..., m_p;\ n_1, n_2, ..., n_q$ are natural numbers and $a_1, a_2, ..., a_p;\ b_1, b_2, ..., b_q$ are real numbers.

Step II:

Here y is zero for x equal to $a_1, a_2, ..., a_p$. These points are marked on the number line with a black dot.

Here y is undefined for x equal to $b_1, b_2, ..., b_q$. These points are marked on the number line with white dot (excluded points in the solution).

Step III:

Check the value of $'y'$ for any real number greater than the right most marked number on the line. If it is positive, then y is positive for all the real numbers greater than the right most marked number and vice versa.

Step IV:

If the exponent of a factor of y is odd, then the point is called simple point and if the exponent of a factor of y is even, then the point is called double point.

Step V:

From right to left beginning above the number line if y is positive in step 3 or from below the line if y is negative in step 3, a wavy curve should be drawn which passes through all the marked points so that when passing through a simple point, the curve intersects the number line and when passing through a double point, the curve remains on the same side of number line.

Step VI:

The intervals where the curve is above number line, y will be positive and the intervals where the curve is below the number line, y will be negative. The appropriate intervals are chosen in accordance with the sign of inequality and their union represents the solution of the inequality.

Practice Problems

Solve the following inequalities:

1. $x^2-3x+2<0$

2. $x^2+5x+6\geq 0$

3. $2x^2+4x+3<0$

4. $x^2+2x+5\geq 0$

5. $(x-1)^2(x-2)^4(x-3)^6\leq 0$

6. $(x-5)^6(x-4)^4(x-3)^9>0$

7. $\dfrac{x-1}{x+2}\geq 0$

8. $\dfrac{x^2+3x+3}{x-2}\leq 0$

9. $\dfrac{x^2-4}{x^2+4}\geq 0$

10. $\dfrac{x-2}{x^2-5x+6}\leq 0$

11. $x^2-4x+3\leq 0$ & $\dfrac{1}{x^2-6x+8}\leq 0$

12. $(x+2)^2<9$

13. $(x-1)^2\geq 4$

14. $x^2+\dfrac{1}{x-2}=4+\dfrac{1}{x-2}$

15. $(x^2+3x+4)(x^2+4x+3)<0$

16. $\dfrac{(x+2)^3(x-3)^4}{x^2-9}\geq 0$

17. $\dfrac{(x-a)^3(x+b)^4}{(x-c)^5}\leq 0$ when b > c > a > 0

18. $\dfrac{1}{x}+\dfrac{1}{x^2}>0$

19. $2^{x^2-5x+2}<\dfrac{1}{4}$

20. $\dfrac{(x^2-6x+8)}{\sqrt{3-x}}\leq 0$

21. $\dfrac{(x-1)(x+2)^2(x-3)}{(x+4)^2(x-5)^4(x+6)^3}\leq 0$

22. $\dfrac{(x-1)^{2019}(x+2)^{2020}}{(x+3)^{2021}(x-4)^{2022}}\geq 0$

23. $1\leq\dfrac{3x^2-7x+8}{x^2+1}\leq 2$

24. $5x-1<(x+1)^2<7x-3$

25. $\dfrac{x+4}{x-2}\leq 1$

26. $\dfrac{1}{x+2}\leq\dfrac{1}{3-x}$

27. $\dfrac{x+2}{2-x}\leq 0$

28. $(x-2)^2(3-x)^3(4-x)^4\leq 0$

29. $\dfrac{(x-a)^1(x-b)^2}{(x-c)^3(x-d)^4}>0$ where $a>b>c>d>0$

30. $\sqrt{x}-3\leq\dfrac{2}{\sqrt{x}-2}$

31. $2+\dfrac{3}{x+1}>\dfrac{6}{x}$

32. $x\leq\dfrac{1}{2-x}$

33. $\sqrt{x^2-4x}>x-2$

34. $\sqrt{2+x-x^2}<x+2$

35. Solve $\dfrac{2(x-3)}{x(x-6)}\leq\dfrac{1}{x-1}$

36. Solve $\dfrac{1}{x-2}+\dfrac{1}{x-1}>\dfrac{1}{x}$

37. Solve $\frac{(x-1)(x-2)(x-3)}{(x+1)(x+2)(x+3)} > 1$

38. Solve $\frac{2(x-4)}{(x-1)(x-7)} \geq \frac{1}{x-2}$

39. Solve $\frac{x^2+2x+5}{(x+1)(x+2)(x-3)} \leq 0$

40. Solve $1 \leq \frac{x^2+5}{x^2+1} \leq 3$

Answers

1.	$(1,2)$	2.	$(-\infty,-3]\cup[-2,\infty)$	3.	ϕ
4.	$\mathbb{R}$	5.	$\{1,2,3\}$	6.	$(3,\infty)-\{4,5\}$
7.	$(-\infty,-2)\cup[1,\infty)$	8.	$(-\infty,2)$	9.	$(-\infty,-2]\cup[2,\infty)$
10.	$(-\infty,3)-\{2\}$	11.	$(2,3]$	12.	$(-5,1)$
13.	$(-\infty,-1]\cup[3,\infty)$	14.	-2	15.	$(-3,-1)$
16.	$(-3,-2]\cup(3,\infty)$	17.	$[a,c)$	18.	$(-1,\infty)-\{0\}$
19.	$(1,4)$	20.	$[2,3)$	21.	$(-\infty,-6)\cup[1,3]$
22.	$(-\infty,-3)\cup[1,\infty)-\{4\}\cup\{-2\}$			23.	$[1,6]$
24.	$(2,4]$	25.	$(-\infty,2)$	26.	$(-\infty,-2)\cup\left[\frac{1}{2},3\right)$
27.	$(-\infty,-2]\cup(2,\infty)$	28.	$[3,\infty)\cup\{2\}$	29.	$(-\infty,c)\cup(a,\infty)-\{d\}$
30.	$[0,1]\cup(4,16]$	31.	$\left(-\infty,\frac{-3}{2}\right)\cup(-1,0)\cup(2,\infty)$	32.	$(-\infty,2)$
33.	$(-\infty,0]$	34.	$[-1,2]$	35.	$(-\infty,0)\cup(1,6)$
36.	$\left(-\sqrt{2},0\right)\cup\left(1,\sqrt{2}\right)\cup(2,\infty)$	37.	$(-\infty,-1)\cup(-2,-3)$	38.	$(1,2)\cup(7,\infty)$
39.	$(-\infty,-1)\cup(-2,3)$	40.	$x\in(-\infty,-1]\cup[1,\infty)$		

Objective Questions

1. If $3-2x \leq 9$ then

(A) $x \leq 8$ (B) $x \geq -6$ (C) $x \leq -3$ (D) $x \geq -3$

2. The solution set for: $-(x-3)+4<5-2x$

(A) $(-\infty, 0)$ (B) $(-\infty, -1)$ (C) $(-\infty, -2)$ (D) $(-\infty, -5)$

3. The solution set for: $\frac{2(x-1)}{5} \leq \frac{3(2+x)}{7}$

(A) $(-24, \infty)$ (B) $[-44, \infty)$ (C) $(-4, \infty)$ (D) none of these

4. Values of x such that $2x+5\le 0$ and $x-3\le 0$

(A) $x\ge -\frac{5}{2}$ (B) $x\le \frac{5}{2}$ (C) $x\le -\frac{5}{2}$ (D) $x\ge \frac{5}{2}$

5. If $\frac{1}{x-2}\ge \frac{1}{3}$; then x belongs to

(A) $(-\infty, 5]$ (B) $[2, 5]$ (C) $(2, 5]$ (D) None of these

6. The solution set of $x^2+8<6x$ is

(A) (2, 6) (B) (4, 8) (C) (6, 8) (D) (2, 4)

7. The solution set of $\frac{x^2-5x+6}{x^2-7x+12}\le 0$

(A) [2, 4) (B) (2, 4) (C) [2, 4) – {3} (D) None of these

8. The solution set of $\frac{x^2+2x+1}{x^2+x+1}\le 0$ is

(A) ϕ (B) $\mathbb{R}$ (C) $\mathbb{R}-\{-1\}$ (D) $\{-1\}$

9. The solution set of $2^{x^2-5x+2}<\frac{1}{4}$

(A) ϕ (B) $\mathbb{R}$ (C) (1, 4) (D) (2, 3)

10. The solution set of $\frac{1}{x}+\frac{1}{x^2}>0$ is

(A) $[-1,\infty)$ (B) $(-1,\infty)$

(C) $(-1,\infty)-\{0\}$ (D) $[-1,\infty)-\{0\}$

11. The solution set of $x^2+\frac{1}{x-2}=4+\frac{1}{x-2}$ is

(A) $\{2,-2\}$ (B) $\{2\}$ (C) $\{-2\}$ (D) None of these

12. The solution set of $\frac{x^2-6x+8}{\sqrt{3-x}}\le 0$ is

(A) $[2,4]$ (B) $[2,3)$ (C) $(3,4]$ (D) $[2,4]-\{3\}$

13. The solution set of $\frac{(x-a)^3(x+b)^4}{(x-c)^2}\le 0$ when $b>c>a>0$ is

(A) $(-\infty,a)$ (B) $(-\infty,b)$ (C) $(-\infty,c)$ (D) none of these

14. The number of integers in the solution set of $(x^2+3x+4)(x^2+4x+3)<0$ is

(A) 0 (B) 1 (C) 3 (D) infinite

15. The solution set of $\frac{(x-a)^1(x-b)^2}{(x-c)^3(x-d)^4} \le 0$ where $a>b>c>d>0$ is

(A) $(c,a]$ (B) $(-\infty,d)\cup[a,\infty)$

(C) $(d,c)\cup[b,a]$ (D) none of these

16. The solution set of $x \le \frac{1}{x-2}$ is

(A) $\left(-\infty,\frac{-8}{5}\right]\cup[2,\infty)$ (B) $\left(-\infty,1-\sqrt{2}\right]\cup\left(2,1+\sqrt{2}\right]$

(C) $(0,3)$ (D) $\left(-\infty,1-\sqrt{2}\right]\cup\left[1+\sqrt{2},\infty\right)$

17. The solution set of $\frac{1}{x+2} \ge \frac{1}{3-x}$ is

(A) $(-2,3)$ (B) $[0,\infty)$ (C) $\left(-2,\frac{1}{2}\right]\cup(3,\infty)$ (D) $(-2,3)-\{0\}$

18. The solution set of $\frac{2x+1}{x^2-x-12} \le \frac{1}{2}$, then $x \in$

(A) $(-\infty,-3)\cup[-2,4)\cup[7,\infty)$ (B) $(-3,-2]\cup(4,7]$

(C) $(-\infty,-2)\cup(3,4)\cup[7,\infty)$ (D) $(-3,-2)\cup(2,4)\cup(7,\infty)$

19. If $\left(x^2-x\right)\left(x^3-x\right)\left(x^4-x\right)<0$, then $x \in$

(A) $(1,1)$ (B) $(-\infty,-1)\cup(0,1)$ (C) $(-\infty,-1)\cup(1,\infty)$ (D) none of these

20. $\frac{x^2+x+1}{x^2+x+2} > 0$ is satisfied by

(A) no value of 'x' (B) all values of 'x'

(C) only positive values of 'x' (D) only negative value of 'x'

KEY

1.	**D**	2.	**C**	3.	**B**	4.	**C**	5.	**C**	6.	**D**
7.	**C**	8.	**D**	9.	**C**	10.	**C**	11.	**C**	12.	**B**
13.	**A**	14.	**B**	15.	**A**	16.	**B**	17.	**C**	18.	**A**
19.	**B**	20.	**B**								

LOGARITHMS

Let 'a' be any positive real number $(a \neq 1)$ and let 'y' be any given positive real number.

If there is a real number 'x' such that $a^x = y$ then x is called logarithm of y to the base a and we write it as $\log_a y$

$\therefore a^x = y \Leftrightarrow \log_a y = x$, $a > 0, a \neq 1, y > 0, x \in \mathbb{R}$

Examples:

1. $2^5 = 32 \Leftrightarrow \log_2 32 = 5$

2. $3^{-4} = \frac{1}{81} \Leftrightarrow \log_3 \frac{1}{81} = -4$

3. $\left(\frac{1}{2}\right)^{-3} = 8 \Leftrightarrow \log_{1/2} 8 = -3$

4. Since $125^{1/3} = 5 \Leftrightarrow \log_{125} 5 = \frac{1}{3}$

Properties of logarithms

Whenever $\log_b a$ is there, $a > 0, b > 0, b \neq 1$

1. $\underset{\text{(exponential form)}}{a^x = y} \Leftrightarrow \underset{\text{(logarthmic form)}}{\log_a y = x}$

2. $a^1 = a \Rightarrow \log_a a = 1$

3. $a^0 = 1 \Rightarrow \log_a 1 = 0$

4. $\log_b a . \log_a b = 1$

5. $\log_b a = \log_b c . \log_c a = \frac{\log_c a}{\log_c b}$

6. $\log_a mn = \log_a m + \log_a n$

7. $\log_a m^n = n \log_a m$

8. $\log_{a^q} m^p = \frac{p}{q} \log_a m$

9. $\log_a \frac{m}{n} = \log_a m - \log_a n$

10. $a^{\log_a n} = n$

11. If $a > 1$, $0 < m < n$ then $\Leftrightarrow$ $\log_a m < \log_a n$

12. If $0 < a < 1$, $0 < m < n$ $\Leftrightarrow$ $\log_a m > \log_a n$

Solutions

4. Let $\log_b a = x$, $\log_a b = y$

$\Rightarrow a = b^x, b = a^y$

$\Rightarrow a = \left(a^y\right)^x = a^{xy}$

$\Rightarrow xy = 1$

$\Rightarrow \log_b a . \log_a b = 1$

5. Let $z = \log_b a$, $x = \log_b c$, $y = \log_c a$

$\Rightarrow a = b^z, c = b^x, a = c^y$

$\Rightarrow a = b^z, a = \left(b^x\right)^y = b^{xy}$

$\Rightarrow b^z = b^{xy}$

$\Rightarrow z = xy$

$\Rightarrow \log_b a = \log_b c . \log_c a$

6. Let $\log_a mn = z, \log_a m = x, \log_a n = y$

$\Rightarrow mn = a^z, m = a^x, n = a^y$

$\Rightarrow mn = a^z, mn = a^x.a^y = a^{x+y}$

$\Rightarrow a^z = a^{x+y}$

$\Rightarrow \log_a mn = \log_a m + \log_a n$

7. Let $\log_a m = x \Rightarrow m = a^x$

$\Rightarrow m^n = a^{nx}$

$\Rightarrow \log_a m^n = nx = n\log_a m$

8. $\log_{a^q} m^p = \dfrac{\log_c m^p}{\log_c a^q} = \dfrac{p\log_c m}{q\log_c a}$

$= \dfrac{p}{q}\log_a m$

9. $\log_a m = x, \log_a n = y$

$\Rightarrow m = a^x, n = a^y$

$\Rightarrow \dfrac{m}{n} = \dfrac{a^x}{a^y} = a^{x-y}$

$\Rightarrow \log_a \dfrac{m}{n} = x - y$

$= \log_a m - \log_a n$

10. Let $\log_a n = x$ ......(1)

$\Rightarrow a^x = n$ ......(2)

Replace x in (2) from (1)

$\Rightarrow a^{\log_a n} = n$

Subjective Questions

1. Find $2^{\log_3 5} - 5^{\log_3 2}$

2. Prove that $\log_{10} 2$ lies between $\dfrac{1}{4}$ & $\dfrac{1}{3}$

3. Prove that $\log_2 3$ is an irrational number

4. If $\dfrac{\log a}{b-c} = \dfrac{\log b}{c-a} = \dfrac{\log c}{a-b}$, prove that $a^a.b^b.c^c = 1$

5. If $\dfrac{\log x}{q-r} = \dfrac{\log y}{r-p} = \dfrac{\log z}{p-q}$, prove that $x^{q+r}.y^{r+p}.z^{p+q} = xyz$

6. If $\log_3 2, \log_3(2x-5)$ and $\log_3\left(2x-\dfrac{7}{2}\right)$ are in A.P, then find the value of 'x'

7. If $\log_{12} 27 = a$ then find $\log_6 16$

8. If $\log_{30} 3 = a$ and $\log_{30}^{5} = b$ then find $\log_{30} 8$

9. If $\log_{12} 18 = a$ and $\log_{24} 54 = b$ prove that $ab + 5(a-b) = 1$

10. If $y = a^{\frac{1}{1-\log_a x}}$ and $z = a^{\frac{1}{1-\log_a y}}$ then prove that $x = a^{\frac{1}{1-\log_a z}}$

Answers

1. 0 6. 4 7. $\dfrac{4(3-a)}{3+a}$ 8. $3(1-a-b)$

Base of a logarithm

There are two important bases in logarithms, they are 10 and e. The logarithms with base 10 are called common logarithms and the logarithms with base 'e' are called natural (Naperian) logarithms. Here 'e' is an irrational number whose value is lying between 2 and 3 $(e \approx 2.71828....)$. Usually $\log x$ means $\log_{10} x$ and $\ln x$ means $\log_e x$.

Characteristic and Mantissa of Logarithmic

If the logarithm of any number is partly integral and partly fractional, the integral portion is called characteristic and decimal part is called mantissa. For example $\log_{10} 200 = 2.3010$ here 2 is called characteristic and 3010 is called mantissa.

The characteristic of a logarithm may be positive, zero or negative but mantissa of logarithm is always a positive real number between 0 and 1 ($0 \le$ mantissa < 1)

For example, we have $\log_{10} \dfrac{1}{2} = \log_{10} 1 - \log_{10} 2 = -0.3010$

Here it looks like characteristic is 0 and mantissa is a negative number.

To express mantissa as a positive number, we write $-0.3010 = -1 + 0.6990 = \bar{1}.6990$

The horizontal line over '1' represents negative, i.e. here characteristic is –1 while mantissa is 0.6990, a positive real number.

Antilogarithm

If $\log_a c = b$, then c is called anti logarithm of b with respect to base a

$\therefore \log_a c = b \Leftrightarrow c = \text{anti } \log_a b$

Thus, the process of finding antilogarithm is just reverse procedure of finding the logarithm of a given number.

Practice Problems

1. Which of the following is FALSE

(A) $\log_5 2 > 0$ (B) $\log_{\frac{1}{5}} 2 < 0$ (C) $\log_5 \dfrac{1}{2} < 0$ (D) $\log_{\frac{1}{5}} \dfrac{1}{2} < 0$

2. Which of the following is FALSE

(A) $\log_2 3 > 1$ (B) $\log_5 4 < 1$ (C) $\log_{\frac{1}{3}} \dfrac{1}{2} < 1$ (D) $\log_{\frac{1}{5}} \dfrac{1}{4} > 1$

3. The value of $2^{2-\log_2 5}$ is

(A) $\dfrac{2}{5}$ (B) $\dfrac{4}{5}$ (C) $\dfrac{5}{2}$ (D) None of these

4. $\log_{5\sqrt{5}} 5 + \log_5 5\sqrt{5}$ is equal to

(A) 1 (B) $\frac{2}{3}$ (C) $\frac{13}{6}$ (D) $\frac{6}{13}$

5. The value of $\log_9 27 - \log_{27} 9$ is

(A) 0 (B) 3 (C) $\frac{1}{3}$ (D) $\frac{5}{6}$

6. The value of $\log_3 4 \log_4 5 \log_5 6 ......\log_{80} 81$ is

(A) 3 (B) 4 (C) 80 (D) 81

7. The value of $16^{\log_4 5} - 4^{\log_{16} 5}$ is

(A) $25-\sqrt{5}$ (B) 0 (C) $5-\sqrt{5}$ (D) None of these

8. If $a^x = b, b^y = c, c^z = a$ then which of the following is FALSE

(A) $x = \log_a b$ (B) $\frac{1}{y} = \log_c b$ (C) $xyz = 1$ (D) $x+y+z=0$

9. If $\log_{0.3}(x-1) < \log_{0.09}(x-1)$ then x lies in the interval

(A) $(2,\infty)$ (B) $(-2,-1)$ (C) $(1,2)$ (D) None of these

10. The value of $\frac{1}{\log_{ab} abc} + \frac{1}{\log_{bc} abc} + \frac{1}{\log_{ca} abc}$ is not equal to

(A) $\log_5 25$ (B) $5^{\log_5 2}$ (C) $\frac{1}{\log_4 2}$ (D) $\log_{abc} abc$

11. If $\frac{\log x}{y-z} = \frac{\log y}{z-x} = \frac{\log z}{x-y}$ then which of the following is FALSE

(A) $xyz = 1$ (B) $x^x y^y z^z = 1$

(C) $x^{y+z}.y^{z+x}.z^{x+y} = 1$ (D) $x+y+z=0$

12. If $(5.6)^a = (0.56)^b = 100$ then $\frac{1}{a} - \frac{1}{b}$ is

(A) 1 (B) $\frac{1}{2}$ (C) $\frac{1}{3}$ (D) 56

13. If a,b,c are distinct natural number such that $\log_2\left(1+\frac{1}{a}\right) + \log_2\left(1+\frac{1}{b}\right) + \log_2\left(1+\frac{1}{c}\right) = 2$ then $a+b+c$ is

(A) 6 (B) 7 (C) 8 (D) 9

14. If x,y,z are three consecutive odd integers then $\log_y(xz+4)$ is

(A) 1 (B) 2 (C) 3 (D) 4

15. If $\log_2 x + \log_4 x + \log_{64} x = 5$ then x is

(A) 8 (B) 16 (C) 32 (D) None of these

16. If $x^2+y^2=6xy$ then $2\log(x+y)-\log xy$ is

(A) $\log 2$ (B) $2\log 2$ (C) $3\log 2$ (D) $4\log 2$

17. If $a=\log_{24}12, b=\log_{36}24, c=\log_{48}36$ then $1+abc$ is equal to

(A) $2ab$ (B) $2bc$ (C) $2ca$ (D) None of these

18. If $\log\left(\frac{a+b}{3}\right)=\frac{\log a+\log b}{2}$ then $\frac{a}{b}+\frac{b}{a}$ is

(A) 2 (B) 3 (C) 6 (D) 7

19. The value of $\frac{1+2\log_3 2}{(1+\log_3 2)^2}+(\log_6 2)^2$ is

(A) 1 (B) 2 (C) 3 (D) 6

20. If $a^8b^9=1$ then $\log_b a^7b^{10}$ is

(A) $\frac{17}{8}$ (B) $\frac{19}{8}$ (C) $\frac{17}{9}$ (D) $\frac{19}{9}$

21. The value of $\frac{60}{\log_4(2000)^6}+\frac{90}{\log_5(2000)^6}$ is

(A) 4 (B) 5 (C) 6 (D) 9

22. Consider the number $\log_{10}2$. It is

(A) A rational number between $\frac{1}{4}$ and $\frac{1}{3}$ (B) An irrational number between $\frac{1}{4}$ and $\frac{1}{3}$

(C) A rational number less than $\frac{1}{4}$ (D) An irrational number less than $\frac{1}{4}$

23. If $\log_p x=a$ and $\log_q x=b$ then $\log_{\frac{p}{q}} x$ is

(A) $\frac{ab}{a-b}$ (B) $\frac{ab}{b-a}$ (C) $\frac{a-b}{ab}$ (D) $\frac{b-a}{ab}$

24. If $\log_a x=6$ and $\log_{25a}8x=3$ then 'a' is

(A) 9 (B) 12.5 (C) 18 (D) 25

25. Let $\log_{12}18=a$. Then $\log_{24}16$ is equal to

(A) $\frac{8-4a}{5-a}$ (B) $\frac{1}{3+a}$ (C) $\frac{4a-1}{2+3a}$ (D) $\frac{8-4a}{5+a}$

KEY

1.	D	2.	D	3.	B	4.	C	5.	D	6.	B
7.	A	8.	D	9.	A	10.	D	11.	D	12.	B
13.	A	14.	B	15.	A	16.	C	17.	B	18.	D
19.	A	20.	A	21.	B	22.	B	23.	B	24.	B
25.	A										