APPLICATIONS OF DERIVATIVES

SINGLE CORRECT OPTION TYPE

1. The least integral value of k for which the function $f(x)=x^2+kx+2018$ is increasing function in the interval $[2019,2020]$ is
(A) –4038 (B) –4040 (C) –4037 (D) –4039

2. The interval of 'x' for which the function $f(x)=-x^2(x-2)$ increases is
(A) $\left(\frac{2}{3},2\right)$ (B) $\left[\frac{2}{3},2\right]$ (C) $\left(0,\frac{4}{3}\right)$ (D) $\left[0,\frac{4}{3}\right]$

3. If the function 'g' defined by $g(x)=f(x^2+2x+8)+f(14-2x-x^2)$, where $f(x)$ is a non-linear twice differentiable function and $f''(x)\geq 0$ for all real numbers x, then the interval in which function $g(x)$ is increasing is
(A) $(-1,1)$ (B) $(3,\infty)$ (C) $(-3,-1)$ (D) $\frac{1}{2}$

4. Which of the following is/are False?
(A) $e^e<\pi^e<e^\pi$ (B) $e^e<e^\pi<\pi^\pi$ (C) $\pi^e<e^\pi<\pi^\pi$ (D) $e^\pi<\pi^e<\pi^\pi$

5. The tangent to the curve y = e^x drawn at the point (c,e^c) intersects the line joining the points $(c-1, e^{c-1})$ and $(c+1, e^{c+1})$)
(A) on the left side of $x=c\ \ \forall c\in R$
(B) on the left side of $x=c$ for only $c>0$
(C) on the right side of $x=c\ \ \forall c\in R$
(D) on the left side of $x=c$ for only c<0

6. If the tangent to the curve $2y^3=ax^2+x^3$ at the point (a,a) cuts off intercepts α and β on the coordinate axes, (where $\alpha^2+\beta^2=61$) then the value of a is
(A) 0 (B) 10 (C) 20 (D) 30

7. If the function $f(x)=ax^3+bx^2+11x-6$ satisfies conditions of Rolle's theorem in [1, 3] and $f'\left(2+\frac{1}{\sqrt{3}}\right)=0$, then value of a and b are respectively.
(A) 1, –6 (B) –1, 6 (C) –2, 1 (D) –1, 1/2

8. If $f(x)=\log_e x$ and $g(x)=x^2$ and $c\in(4,5)$, then $c\log\left(\frac{4^{25}}{5^{16}}\right)$ is equal to
(A) $c\log_e 5-8$ (B) $2(c^2\log_e 4-8)$ (C) $2(c^2\log_e 5-8)$ (D) $c\log_e 4-8$

9. If the function $f(x)=x^3-9x^2+24x+c$ has three real and distinct roots, α, β and γ, then the value of $[\alpha]+[\beta]+[\gamma]$ is
(A) 5, 6 (B) 6, 7 (C) 7, 8 (D) 9, 10

10. The values of 'K' for which the point of minimum of the function $f(x)=1+K^2x-x^3$ satisfy the inequality $\frac{(x^2+x+2)}{(x^2+5x+6)}<0$, belongs to
(A) $(-3\sqrt{3},\infty)$
(B) $(-3\sqrt{3},-2\sqrt{3})\cup(0,\infty)$
(C) $(-3\sqrt{3},-2\sqrt{3})\cup(2\sqrt{3},3\sqrt{3})$
(D) $(0,\infty)$

11. Suppose that f is differentiable for all x, such that $f'(x) \le 2$ for all x. If $f(1)=2$ and $f(4)=8$ then f(2) has the value equal to

(A) 10 (B) 4 (C) 5 (D) 3

12. Let y=f(x) be a thrice differentiable function defined on R such that f(x)=0 has at least 5 distinct zeros, then minimum number of zeros of the equation $f(x)+6f'(x)+12f''(x)+8f'''(x)=0$ is

(A) 5 (B) 2 (C) 3 (D) 4

13. Let f(x) and g(x) be two differentiable function in R and f(2)=8, g(2)=0, f(4)=10 and g(4)=8 then which of the following is correct?

(A) $g'(x) > 4f(x) \forall x \in (2,4)$

(B) $3g'(x) = 4f'(x)$ for at least one $x \in (2,4)$

(C) $g(x) > f(x) \forall x \in (2,4)$

(D) $g'(x) = 4f'(x)$ for at least one $x \in (2,4)$

14. Let a, x and y be real numbers such that $0 < a < 1$ and $x^2 + y = 0$. The maximum value of $\log_a(a^x + a^y)$ is

(A) $4\log_a^2 + 6$ (B) $\log_a\left(\frac{1}{2}\right) + 8$ (C) $2\log_a^2 + \frac{1}{6}$ (D) $\frac{1}{8} + \log_a^2$

15. Let $f(x) = x^3 + x^2 + 2x - 1$. The minimum value of [x] that satisfy the $f(f(x)) > f(2x+1)$ is______. (Where [.] denotes the greatest integer function).

(A) 0 (B) 1 (C) 2 (D) 3

16. If $f'(x) > 0, \forall x \in R, f'(3) = 0$ and $g(x) = f(\tan^2 x - 2\tan x + 4), 0 < x < \frac{\pi}{2}$, then g(x) increases in

(A) $(0,\pi/4)$ (B) $(\pi/4, \pi/2)$ (C) $(\pi/4, \pi/3)$ (D) $(0,\pi/3)$

17. If the function $f(x) = 2x^3 - 9ax^2 + 12a^2x + 1$, where a>0, attains its maximum and minimum at p and q respectively such that $q^2 = q$ then a equals:

(A) 3 (B) 1 (C) 2 (D) 1/2

18. The maximum value of $\left(\sqrt{-3+4x-x^2}+4\right)^2 + (x-5)^2$. (Where $1 \le x \le 3$) is

(A) 34 (B) 36 (C) 32 (D) 20

19. If the equation $2x^3 - 3x^2 - 12x + k = 0$ has 3 real roots, then

(A) $-7 < k < 7$ (B) $-\infty < k < 1$ (C) $-7 < k < 20$ (D) $-1 < k < 7$

20. In a rhombus of side length 5, the length of one of the diagonal is at least 6, and the length of the other is at most 6. What is the maximum value of the sum of the lengths of diagonals?

(A) $5\sqrt{6}$ (B) 14 (C) $10\sqrt{2}$ (D) 12

NUMERICAL BASED

21. For a given function $f(x)$, which have domain and co-domain as real number set is defined by $f(-x) = x^3 e^{-x}$ is decreasing at 'm' non-positive integer(s). Then the number of proper divisor(s) of sum of divisors of 'm' will be ___

22. Given 2 functions $f,g: R \to R$, where $f(x) = \frac{x^{2019}}{2019} - \frac{x^{2017}}{2017}$ and $g(x) = \frac{x^{2020}}{2020} - \frac{x^{2018}}{2018}$. The number of non-negative integer(s) at which $f(x), g(x)$ are strictly decreasing are λ and μ respectively. Then $\lambda + \mu + \lambda\mu =$ ___

23. Given that a function $f(x)$ satisfies the relation for $f\left(\frac{x-3}{x+1}\right)+f\left(\frac{3+x}{1-x}\right)=x$ all real numbers x such that $|x|\neq 1.$ Number of points on $y=f(x)$ tangent at which is parallel to the line $7x-2y=15$ is

24. The least value of for which the equation, $\frac{4}{\sin x}+\frac{1}{1-\sin x}=a$ has atleast one solution on the interval (0, π/2) is ………..

25. Given that $S=\left|\sqrt{x^2+4x+5}-\sqrt{x^2+2x+5}\right|$ for all real *x*, then the maximum value of S^4 is

KEY

1.	C	**2.**	C	**3.**	C	**4.**	D	**5.**	A
6.	D	**7.**	A	**8.**	B	**9.**	C	**10.**	D
11.	B	**12.**	B	**13.**	D	**14.**	D	**15.**	B
16.	B	**17.**	C	**18.**	B	**19.**	C	**20.**	B
21.	1	**22.**	1	**23.**	3	**24.**	9	**25.**	4

❈ *Wish You all the Best* ❈