DEFINITE INTEGRATION

Single Correct Answer Type:

1. Given that $\lim_{n\to\infty}\sum_{r=1}^{n}\frac{\log\left(n^2+r^2\right)-2\log n}{n}=\log 2+\frac{\pi}{2}-2$, then $\lim_{n\to\infty}\frac{1}{n^{2m}}\left[\left(n^2+1^2\right)^m\left(n^2+2^2\right)^m.........\left(2n^2\right)^m\right]^{1/n}$ is equal to

(A) $2^m e^{m(\pi/2-2)}$ (B) $2^m e^{m(2-\pi/2)}$ (C) $e^{m(\pi/2-2)}$ (D) $e^{2m(\pi/2-2)}$

2. The value of $\int_{-6}^{6}\max\left(\left|2-|x|\right|,\ 4-|x|,\ 3\right)dx$ is

(A) 40 (B) 50 (C) 60 (D) 30

3. If $I_n=\int_0^{\pi}e^x(\sin x)^n\,dx$, then $\frac{I_3}{I_1}$ is equal to

(A) $\frac{3}{5}$ (B) $\frac{1}{5}$ (C) 1 (D) $\frac{2}{5}$

4. If $m=\int_{-2}^{0}\frac{|\sin x|}{\left[\frac{x}{\pi}\right]+\frac{1}{2}}dx$ and $n=\int_{0}^{2}\frac{|\sin x|}{\left[\frac{x}{\pi}\right]+\frac{1}{2}}dx$, where [.] represents greatest integer function, then

(A) $m=n$ (B) $m=-n$ (C) $m=2n$ (D) $m=-2n$

5. Let $\lambda=\int_0^1\frac{dx}{1+x^3}$, $p=\lim_{n\to\infty}\left[\frac{\prod_{r=1}^{n}\left(n^3+r^3\right)}{n^{3n}}\right]^{1/n}$, then $\ell n\,p$ is equal to

(A) $\ell n2-1+\lambda$ (B) $\ell n2-3+3\lambda$ (C) $2\ell n2-\lambda$ (D) $\ell n4-3+3\lambda$

6. $\int_0^{\pi/2}\frac{4x\sin x+x^2\cos x}{2\sqrt{\sin x}}dx$ is equal to

(A) $\frac{\pi}{2}$ (B) $\frac{\pi^2}{4}$ (C) $\frac{\pi}{4}$ (D) $\frac{\pi^2}{16}$

7. $\int_0^{\infty}\frac{x\log x\,dx}{\left(1+x^2\right)^2}=$

(A) 0 (B) $\frac{\pi}{2}$ (C) $\frac{\pi}{4}$ (D) 1

8. $\int_{-2}^{2}\max\left(x+|x|,\ x-[x]\right)dx$ (where [x] denotes the greatest integer $\le x$) is equal to

(A) 4 (B) 5 (C) 6 (D) 8

9. If $y=\int_1^x x^2\sqrt{\ell n\,t}\,dt$; then the value of $\frac{d^3y}{dx^3}$ at $x=e$

(A) 33 (B) 11 (C) $\frac{33}{4}$ (D) None of these

10. If $\int_0^{\infty}\left(\frac{\sin x}{x}\right)^3 dx = A$ and $\int_0^{\infty}\left(\frac{x-\sin x}{x^3}\right)dx = \frac{aA}{b}$, where a ad b are relative prime then the value of $(a+b)$ equals

(A) 3 (B) 4 (C) 5 (D) 6

11. If $f(x) = x^3 + 3x + 4$ then the value of $\int_{-1}^{1} f(x)dx + \int_0^4 f^{-1}(x)dx$ equals

(A) 4 (B) $\frac{17}{4}$ (C) $\frac{21}{4}$ (D) $\frac{23}{4}$

12. Let $I(n) = \int_{2016}^{2016+\frac{1}{n}} x\cos^2(x-2016)dx$,

(a) $I\left(\frac{1}{\pi}\right) = \frac{(\pi+2016)\pi}{2}$ (b) $I\left(\frac{1}{\pi}\right) = \frac{(\pi+4032)\pi}{4}$ (c) $\lim_{n\to\infty} n.I(n) = 2016$ (d) $\lim_{n\to\infty} n.I(n) = 2017$

then which of the above are correct?

(A) a, c (B) a, d (C) b, c (D) b, d

13. Let $f: D \to y$, $f(x) = \ell n\left[\cos|x| + \frac{1}{2}\right]$, where [.] represents greatest integer function) then $\int_{x_1}^{x_2}\left(\lim_{n\to\infty}\left(\frac{(f(x))^n}{x^2+\tan^2 x}\right)\right)dx$ where $(x_1, x_2 \in D)$ is

(A) 0 (B) 1 (C) $\frac{3}{2}$ (D) 2

14. Let $f(x) = \int_1^x \left(2(x-1)(x-2)^3 + 3(x-1)^2(x-2)^2\right)dx$, then

(A) f has exactly 3 points for local extremum

(B) f has local minima at x = 2 and local maxima at $x = \frac{7}{5}$

(C) $x = \frac{7}{5}$ is local minima & x = 1 is local maxima

(D) None of these

15. Let $I_1 = \int_0^1 \frac{e^x dx}{x+1}$ and $I_2 = \int_0^1 \frac{x^2 dx}{e^{x^3}(2-x^3)}$, then $\frac{I_1}{I_2}$ is equal to

(A) $\frac{3}{e}$ (B) $\frac{e}{3}$ (C) 3e (D) $\frac{1}{3e}$

16. The value of $\int_0^{\infty}\frac{(x^2+4)\ln x\,dx}{(x^4+16)}$ is equal to

(A) $\frac{\pi}{2\sqrt{2}}\ln 2$ (B) $\frac{\pi}{2}\ln 2$ (C) $\pi\ln 2$ (D) $\frac{\pi}{\sqrt{2}}\ln 2$

17. If $f(x)$ is differentiable function such that $f'(x)$ is symmetrical about $2x = 3$ and $f(1) = 0$, $f(2) = 8$ then $\int_0^2 f(x)[x]dx$ is (where [.] represent G.I.F.)

(A) 4 (B) 8 (C) 0 (D) cannot be determined

18. The value of $\lim_{n\to\infty} \int_0^{(n!+1)\pi/2} \left(\frac{\cos x + x\sin x}{x^2 + \cos^2 x}\right)dx$ is (n > 1)

(A) $\frac{\pi}{2}$ (B) $-\frac{\pi}{2}$ (C) 0 (D) π

19. The value of $\sum_{r=2}^{16} \int_r^{r+1} \frac{dx}{(2r - x)(2r + 2 - x)}$ is equal to

(A) $\ell n\left(\frac{4}{3}\right)$ (B) $\ell n\left(\frac{2}{3}\right)$ (C) $\tan^{-1}\left(\frac{2}{3}\right)$ (D) $\ell n\left(\frac{2\sqrt{2}}{3}\right)$

20. If $f(x)$ is continuous function such that $f(x) > 0\ \forall\ x \geq 0$ and $(f(x))^{2014} = 1 + \int_0^x f(t)dt$ then the value of $(f(2014))^{2013}$ is equal to

(A) 2013 (B) 2014 (C) 2015 (D) 2016

Numerical Based:

21. If $\int_0^\infty \left(\frac{\ln x}{1-x}\right)^2 dx + \int_0^1 k\frac{\ln(1-x)}{x}dx = 0$ then k is ________

22. Let $f(x) = \lim_{n\to\infty} \frac{\cos x}{1 + (\tan^{-1} x)^n}$ and $\int_0^\infty f(x)dx = \sin(\tan\theta)$, then θ is

23. $\int_{\pi/4}^{\pi/2} \frac{dx}{\left(\sin x + \cos x + 2\sqrt{\sin x.\cos x}\right)\sqrt{\sin x.\cos x}}$ is

24. Given that $\lim_{n\to\infty} \int_0^\infty \cos\frac{x}{2}\cos\frac{x}{4}\cos\frac{x}{8}........\cos\frac{x}{2^{n-1}}dx = \frac{\pi}{2}$, then the value of $\int_0^\infty \frac{\sin^3 x}{x}dx = \frac{\pi}{\lambda}$. Find λ.

25. Let f be a function defined by $f(x) = \frac{\lambda^x}{\lambda^x + \sqrt{\lambda}}(\lambda > 0)$ and $I_1 = \int_{f(1-k)}^{f(k)} x f(x(1-x))dx$ and $I_2 = \int_{f(1-k)}^{f(k)} f(x(1-x))dx$, where $2k - 1 > 0$, if $\frac{I_1}{I_2} = \frac{a}{2}$, then a is equal to

KEY

1.	A	2.	A	3.	A	4.	B	5.	B
6.	B	7.	A	8.	B	9.	C	10.	C
11.	D	12.	C	13.	A	14.	C	15.	C
16.	A	17.	A	18.	A	19.	D	20.	B
21.	1	22.	4	23.	1	24.	4	25.	1