SINGLE CORRECT OPTION TYPE

1. The number of true statements in the following list.

S_1: If $f(x) < g(x)$, then $f'(x) < g'(x)$

S_2: If f is derivable on $[a,b]$, $f(a) = f(b) = 0$ and $f(x) \neq 0$ for any $x \in (a,b)$ then $f'(a)$ and $f'(b)$ must be of opposite sign.

S_3: Suppose $y = f(x)$ and $x = g(y)$ are inverse functions of each other such that

$$g'''(y) = \frac{f'(x)f'''(x) + \alpha(f''(x))^{\delta}}{(f'(x))^{\beta}} \text{ then } \beta + \alpha - \delta = 0$$

S_4: If $y = x^{2018}(x-1)^4(x-2)^3(x-3)^2$ then $y''''(1) + 8\alpha = 0$ where α is divisible by

$$\beta = \underset{x\to 0}{\mathrm{Lt}}\left[\frac{3\sin x}{x}\right] + \left[\frac{4x}{\tan x}\right].$$

Here [.] denotes the greatest integer function and y'''' denote the 4^{th} order derivative if y.

(f', f'', f'''......... denotes the first order , second order, third derivatives respectively)

(A) 2 (B) 1 (C) 4 (D) 3

2. If $f(x) = (1-x)^{-1}, |x| < 1$ then the value of $\frac{f^{ii}(0)}{f(0)} + \frac{f^{iii}(0)}{f^{i}(0)} + \frac{f^{iv}(0)}{f^{ii}(0)} + ... + \frac{f^{n+1}(0)}{f^{n-1}(0)}$ is

(A) $\frac{n(n+1)(n+2)}{3}$ (B) $\frac{n(n+1)(2n+1)}{6}$ (C) $\frac{n(n+1)(n+2)}{6}$ (D) $\frac{n(n+1)(2n+1)}{3}$

3. $\frac{d}{dx}\left(\log\left(e^x\left(\frac{x-2}{x+2}\right)^{3/4}\right)\right)$ Equal to

(A) $\frac{x^2-1}{x^2-4}$ (B) 1 (C) $\frac{x^2+1}{x^2-4}$ (D) $e^x\left(\frac{x^2-1}{x^2-4}\right)$

4. $\frac{d}{dx}\left(\sin^{-1}\left(\frac{2}{x^{-1}+x}\right)\right)$ is equal to

(A) $\frac{2}{1+x^2}, 0 < |x| < 1$ (B) $\frac{2}{\sqrt{1-x^2}}, |x| < 1$ (C) $\frac{2}{\sqrt{1+x^2}}$ (D) 0

5. If $y = \sin^{-1}\left(\frac{\sqrt{x}-1}{\sqrt{x}+1}\right) + \sec^{-1}\left(\frac{\sqrt{x}+1}{\sqrt{x}-1}\right)$, $x > 0$ then $\frac{dy}{dx} =$

(A) 1 (B) 0 (C) $\frac{\pi}{2}$ (D) -1

6. If $f(x) = \cot^{-1}\left(\frac{x^x - x^{-x}}{2}\right)$ then $f^1(1) =$

(A) $-\ln 2$ (B) $\ln 2$ (C) 1 (D)-1

7. If $f(x)$ is a function such that $f''(x) + f(x) = 0$ and $g(x) = (f(x))^2 + (f'(x))^2$ and $g(3) = 3$ then $g(8) =$

(A) 5 (B) 0 (C) 3 (D) 8

8. If f,g,h are such that $f^1(x)=g(x), g^1(x)=h(x)$, $h^1(x)=f(x), f(0)=1, g(0)=0$ and $h(0)=0$ then the value of $f^3(x)+g^3(x)+h^3(x)-3f(x)\ g(x)\ h(x)$ at $x=5$ is

(A) 0 (B) 1 (C) 2 (D) 3

9. If a curve is represented parametrically by the equations $x=\sin\left(t+\frac{7\pi}{12}\right)+\sin\left(t-\frac{\pi}{2}\right)+\sin\left(t+\frac{3\pi}{12}\right)$, $y=\cos\left(t+\frac{7\pi}{12}\right)+\cos\left(t-\frac{\pi}{2}\right)+\cos\left(t+\frac{3\pi}{12}\right)$, then the value of $\frac{d}{dt}\left(\frac{x}{y}-\frac{y}{x}\right)$ at $t=\frac{\pi}{8}$ is

(A) 0 (B) 4 (C) 8 (D) 12

10. If $x^2+y^2+z^2-2xy=1$ and $\frac{dx}{\sqrt{1-x^2}}+\frac{dy}{\sqrt{1-y^2}}+\frac{dz}{\sqrt{1-z^2}}=3+k$, then $k=$

(A) 0 (B) 3 (C) –3 (D) –2

11. If $f(x)=\lim_{n\to\infty}\left(\prod_{i=1}^{n}\cos\left(\frac{x}{2^i}\right)\right)$, then $f'(x)$ is equal to

(A) $\frac{\sin x}{x}$ (B) $\frac{x}{\sin x}$ (C) $\frac{x\cos x-\sin x}{x^2}$ (D) $\frac{\sin x-x\cos x}{\sin^2 x}$

12. Let $f(\alpha)=\cos\left[\cot^{-1}\left(\frac{\cos\alpha}{\sqrt{1-\cos 2\alpha}}\right)\right]$ where $\frac{\pi}{4}<\alpha<\frac{\pi}{2}$ then the value of $\frac{df(\alpha)}{d(\cot\alpha)}$ is

(A) 6 (B) 5 (C) 3 (D) 1

13. If $y=\sin x+e^x$, then $\frac{d^2x}{dy^2}$

(A) $\left(-\sin x+e^x\right)^{-1}$ (B) $\frac{\sin x-e^x}{\left(\cos x+e^x\right)^3}$ (C) $\frac{\sin x-e^x}{\left(\cos x+e^x\right)^2}$ (D) $\frac{\sin x+e^x}{\left(\cos x+e^x\right)^3}$

14. Let $f(x)=\exp\left(x^3+x^2+x\right)$ for any real number x and let g be the inverse of f, then the value of $g'\left(e^3\right)$ is

(A) $\frac{1}{6}$ (B) 6 (C) $\frac{1}{6e^3}$ (D) $6e^3$

15. Let $x,y\in\mathbb{R}$, satisfying the equation $\cot^{-1}x+\cot^{-1}y+\cot^{-1}(xy)=\frac{11\pi}{12}$ then the value of $\frac{dy}{dx}$ at $x=1$ is

(A) $-\left(\frac{3+\sqrt{3}}{3}\right)$ (B) $\left(\frac{1}{3}+\frac{1}{2\sqrt{3}}\right)$ (C) $-\left(\frac{5+\sqrt{3}}{3}\right)$ (D) $-\left(\frac{1}{3}+\frac{1}{2\sqrt{3}}\right)$

16. Let $f:R\to R$ defined as $f(x)=x^3+3x^2+6x-5+4e^{2x}$ and $g(x)=f^{-1}(x)$ then $g'(-1)=$

(A) $\frac{1}{7}$ (B) $\frac{1}{10}$ (C) $\frac{1}{14}$ (D) $\frac{1}{17}$

17. If $y=\sqrt{(a-x)(x-b)}-(a-b)\tan^{-1}\sqrt{\frac{a-x}{x-b}}$ then $\frac{dy}{dx}$

(A) $\sqrt{(a-x)(x-b)}$ (B) $\sqrt{\frac{a-x}{x-b}}$ (C) $\sqrt{a-x}$ (D) $\sqrt{x-b}$

18. If $f(x)=2\sin^{-1}\sqrt{1-x}+\sin^{-1}\left(2\sqrt{x(1-x)}\right)$, where $x\in\left(0,\frac{1}{2}\right)$ then $f'(x)$ is

(A) $-\frac{2}{\sqrt{x(1-x)}}$ (B) $\frac{2}{\sqrt{x(1-x)}}$ (C) 0 (D) None

19. Suppose $f(x)=e^{ax}+e^{bx}$, where $a\neq b$, and that $f''(x)-2f'(x)-15f(x)=0$ for all x. Then the product ab equal to

(A) 25 (B) 9 (C) –15 (D) –9

20. The equation $y^2e^{xy}=9e^{-3}x^2$ defines y as a differentiable function of x. The value of $\frac{dy}{dx}$ for $x=-1$ is

(A) $-\frac{15}{2}$ (B) $-\frac{9}{5}$ (C) 3 (D) 15

NUMERICAL BASED

21. If $y=\frac{x^4+x^2+1}{x^2+x+1}$ such that $\frac{dy}{dx}=ax+b$ then $a^2+b^2=$

22. Let $f(x)=(x-1)(x-2)(x-3)...(x-n), n\in N$ and $f^1(n)=5040$ then the value of n is

23. Suppose the function $f(x)-f(2x)$ has the derivative 5 at $x=1$, and derivative 7 at $x=2$. The derivative of function $f(x)-f(4x)$ at $x=1$, has the value equal to

24. Let $f(x)=x+\cfrac{1}{2x+\cfrac{1}{2x+\cfrac{1}{2x+........\infty}}}$ then $f(7).f'(7)$ is

25. If 'f' be a twice differentiable such that $f''(x)=-f(x)$ and $f'(x)=g(x)$ if $h(x)=\{f(x)\}^2+\{g(x)\}^2$ where $h(5)=11$, then $h(2015)=$ ______

KEY

1.	A	**2.**	A	**3.**	A	**4.**	A	**5.**	B
6.	D	**7.**	C	**8.**	B	**9.**	C	**10.**	C
11.	C	**12.**	D	**13.**	B	**14.**	C	**15.**	D
16.	C	**17.**	B	**18.**	C	**19.**	C	**20.**	D
21.	5	**22.**	8	**23.**	14	**24.**	7	**25.**	11

❉ *Wish You all the Best* ❉