INDEFINITE INTEGRATION

Single Correct Answer Type:

1. Let $\int \tan^{-1}\left(\frac{\tan x}{2}\right)dx = \alpha$ then the value of $\int \tan^{-1}\left(\frac{\tan x - 2\cot x}{3}\right)dx$ equals to: $\left(0 < x < \frac{\pi}{2}\right)$

(A) $\alpha + \frac{\pi x}{2} + \frac{x^2}{2} + c$ (B) $\alpha - \frac{\pi x}{2} + \frac{x^2}{2} + c$ (C) $-\alpha - \frac{\pi x}{2} + \frac{x^2}{2} + c$ (D) $-\alpha + \frac{\pi x}{2} - \frac{x^2}{2} + c$

2. Let $f(x) = \lim_{n\to\infty} n^2\left(\frac{1}{x^n} - \frac{1}{x^{n+1}}\right)$; $x > 0$, then $\int x f(x)dx$ equals to

(A) $\frac{x^2}{2}\ell nx + c$ (B) $-\frac{x^2}{4}\ell nx + \frac{x^2}{2} + c$ (C) $\frac{x^2}{2}\ell nx + \frac{x^2}{4} + c$ (D) $\frac{x^2}{2}\ell nx - \frac{x^2}{4} + c$

3. $\int\left(\int e^x\left(\ln x + \frac{2}{x} - \frac{1}{x^2}\right)dx\right)dx =$

(A) $e^x \ln x + C_1x + C_2$ (B) $e^x \ln x + \frac{1}{x} + C_1x + C_2$ (C) $\frac{\ln x}{x} + C_1x + C_2$ (D) None of these

4. Let $F(x) = \int e^{\sin^{-1}x}\left(1 - \frac{x}{\sqrt{1-x^2}}\right)dx$ and $F(0) = 1$. If $F\left(\frac{1}{2}\right) = \frac{k\sqrt{3}e^{\frac{\pi}{6}}}{\pi}$, then k is equal to

(A) 2π (B) π (C) $3\pi/2$ (D) $\pi/2$

5. $\int \frac{1}{(\tan x + 1)\sin^2 x}dx = f(x) + c$, where $f\left(\frac{\pi}{2}\right) = 0$ then the value of $f\left(\frac{\pi}{4}\right)$ is

(A) $1 - \ell n2$ (B) $\ell n2 - 1$ (C) $1 + \ell n2$ (D) $-1 - \ell n2$

6. $\int \frac{x^2(1 - \ell nx)}{\ell n^4 x - x^4}dx$ equals

(A) $\frac{1}{2}\ell n\left(\frac{x}{\ell nx}\right) - \frac{1}{4}\ell n\left(\ell n^2 x - x^2\right) + C$

(B) $\frac{1}{4}\ell n\left(\frac{\ell nx - x}{\ell nx + x}\right) - \frac{1}{2}\tan^{-1}\left(\frac{\ell nx}{x}\right) + C$

(C) $\frac{1}{4}\ell n\left(\frac{\ell nx + x}{\ell nx - x}\right) + \frac{1}{2}\tan^{-1}\left(\frac{\ell nx}{x}\right) + C$

(D) $\frac{1}{4}\left(\ell n\left(\frac{\ell nx - x}{\ell nx + x}\right) + \tan^{-1}\left(\frac{\ell nx}{x}\right)\right) + C$

7. $\int \frac{(x - \sin x)^{3/2}}{\sqrt{x}}\left\{\frac{6x^2 \sin^2\left(\frac{x}{2}\right)}{x - \sin x} + 3x\right\}dx$ is equal to

(A) $x^{3/2}(x - \sin x)^{3/2} + C$

(B) $2x^{3/2}(x - \sin x)^{3/2} + C$

(C) $x^{1/2}(x - \sin x)^{3/2} + C$

(D) $2x^{1/2}(x - \sin x)^{3/2} + C$

8. Let $I_{(m,n)} = \int (\cos 2x)^m \left(1 - \tan^2 x\right)^n dx$, such that $4I_{(0,10)} + \frac{I_{(1,10)}}{5} = A\sin 2x\left(1 - \tan^2 x\right) + BI_{(0,9)} + C$ (where C is integration constant), then value of $\frac{B}{8A}$ is

(A) 2 (B) 3 (C) 4 (D) 5

9. Let $f(x)=\int \frac{x^2+4}{x^4+x^3-7x^2-4x+16}dx=\frac{A}{\sqrt{3}}\tan^{-1}\left(\frac{Ax^2+Bx+2C}{\sqrt{3}x}\right)$ such that $f(2)=\frac{\pi}{3\sqrt{3}}(A,B,C\in R)$, the value of $|A+B+C|$ is

(A) 1 (B) 2 (C) 3 (D) 4

10. If $\int e^{1-\cos x}(1+x\sin x)dx=f(x).e^{1-g(x)}+c$ then number of solutions of the equation $|f(x)|=g(x)$ equals

(A) 0 (B) 1 (C) 2 (D) 3

11. If $\int \frac{x-\sin x\cos x}{x^2\cos^2 x}dx=f(x)+c$ then $\lim_{x\to 0} f(x)$ equals

(A) 1 (B) $\sqrt{2}$ (C) $\frac{1}{\sqrt{2}}$ (D) $\sqrt{3}$

12. Let $\int \frac{x^2(x\sec^2 x+\tan x)}{(x\tan x+1)^2}dx=2\ell n|f(x)|-\frac{x^2}{g(x)}+k$ (where k is constant of integration)

$\lim_{x\to 0}\frac{g(x)-1}{f(x)-1}$ is equal to

(A) 0 (B) 1 (C) 2 (D) does not exist

13. If $\int \frac{(2x+3)dx}{x(x+1)(x+2)(x+3)+1}=C-\frac{1}{f(x)}$ where $f(x)$ is of the form of ax^2+bx+c then $(a+b+c)$ equals

(A) 4 (B) 5 (C) 6 (D) None of these

14. Primitive of $f(x)=x.2^{\ell n(x^2+1)}$ w.r.t. x is

(A) $\frac{2^{\ell n(x^2+1)}}{2(x^2+1)}+C$ (B) $\frac{(x^2+1)2^{\ell n(x^2+1)}}{\ell n2+1}+C$ (C) $\frac{(x^2+1)^{\ell n2+1}}{2(\ell n2+1)}+C$ (D) $\frac{(x^2+1)^{\ell n2}}{2(\ell n2+1)}+C$

15. Which of the following is TRUE?

(A) $x.\int\frac{dx}{x}=x\ell n|x|+C$ (B) $x.\int\frac{dx}{x}=x\ell n|x|+Cx$

(C) $\frac{1}{\cos x}.\int\cos x\,dx=\tan x+C$ (D) $\frac{1}{\cos x}.\int\cos x\,dx=x+C$

16. The evaluation of $\int \frac{px^{p+2q-1}-qx^{q-1}}{x^{2p+2q}+2x^{p+q}+1}dx$ is

(A) $-\frac{x^p}{x^{p+q}+1}+C$ (B) $\frac{x^q}{x^{p+q}+1}+C$ (C) $-\frac{x^q}{x^{p+q}+1}+C$ (D) $\frac{x^p}{x^{p+q}+1}+C$

17. If $I_n=\int(\sin x)^n dx, n\in N$, then $5I_4-6I_6$ is equal to

(A) $\sin x(\cos x)^5+C$ (B) $\sin 2x.\cos 2x+C$

(C) $\frac{\sin 2x}{8}\left[\cos^2 2x+1-2\cos 2x\right]+C$ (D) $\frac{\sin 2x}{8}\left[\cos^2 2x+1+2\cos 2x\right]+C$

18. Primitive of $f(x)=\frac{ax^2+2bx+c}{(Ax^2+2Bx+C)^2}$ w.r.t. x (where $B^2=AC$)

(A) has a term containing logarithmic function
(B) has a tem containing inverse function
(C) has terms containing both logarithmic and inverse functions
(D) is a rational function

19. $\int \sec^2\theta(\sec\theta+\tan\theta)^2 d\theta =$

(A) $\frac{(\sec\theta+\tan\theta)}{2}\left[2+\tan\theta(\sec\theta+\tan\theta)\right]+C$ (B) $\frac{(\sec\theta+\tan\theta)}{3}\left[2+4\tan\theta(\sec\theta+\tan\theta)\right]+C$

(C) $\frac{(\sec\theta+\tan\theta)}{3}\left[2+\sec\theta(\sec\theta+\tan\theta)\right]+C$ (D) $\frac{3(\sec\theta+\tan\theta)}{2}\left[2+\tan\theta(\sec\theta+\tan\theta)\right]+C$

20. Suppose $J=\int\frac{\sin^2 x+\sin x}{1+\sin x+\cos x}dx$ and $K=\int\frac{\cos^2 x+\cos x}{1+\sin x+\cos x}dx$. If C is an arbitrary constant of integration then which of the following is/are correct?

(A) $J=\frac{1}{2}(x-\sin x+\cos x)+C$ (B) $J=K-(\sin x+\cos x)+C$

(C) $J=2x-K+C$ (D) $K=\frac{1}{2}(x-\sin x+\cos x)+C$

Numerical Based:

21. Let $f(x)=\int(x\cos x+1)e^{\sin x}dx$ and $g(x)=\int(x\sin x-1)e^{\cos x}dx$. Further suppose that $f(0)=g(0)=0$. Find the number of distinct solution (s) of the equation $f(x).g(x)=0$.

22. Let $f(x)$ be a differentiable function satisfying the equation $\frac{f'(x)}{2}=\frac{x}{e^{f(x)}}\forall x\in R$. If $f'(1)=1$; find the number of solution of the equation $f(x)=f'(x)$

23. $\int\frac{\left(1+x^{1/4}\right)^{1/3}}{\sqrt{x}}dx=\frac{a}{b}\left(1+x^{1/4}\right)^{7/3}-c\left(1+x^{1/4}\right)^{4/3}+k$; a & b are coprime. Find value of $(a-b+c)$

24. If $\int\frac{\left(3x^2+2x\right)}{\left(x^6+2x^5+x^4+2x^3+2x^2+5\right)}dx=Af\left(\frac{g(x)}{B}\right)+c$ where c is integration constant then find the value of $\frac{B}{A}$.

25. If $F(x)=\int\frac{(1+x)\left(\left(1-x+x^2\right)\left(1+x+x^2\right)+x^2\right)}{1+2x+3x^2+4x^3+3x^4+2x^5+x^6}dx$ then find the value of $\left[F(99)-F(3)\right]$. (where [.] denotes greatest integer function.

KEY

1.	B	2.	D	3.	A	4.	D	5.	B
6.	B	7.	B	8.	D	9.	A	10.	C
11.	A	12.	A	13	B	14.	C	15.	B
16.	C	17.	C	18.	A	19.	C	20.	B
21.	1	22.	2	23.	8	24.	4	25.	3