# HEAT AND THERMODYNAMICS



**Single Correct Answer Type:**

1. A substance of mass *m* kg requires a power input of *P* watts to remain in the molten state at its melting point. When the power is turned off, the sample completely solidifies in time *t* sec. What is the latent heat of fusion of the substance

(A) $\frac{Pm}{t}$ (B) $\frac{Pt}{m}$ (C) $\frac{m}{Pt}$ (D) $\frac{t}{Pm}$

2. A lead bullet at 27°*C* just melts when stopped by an obstacle. Assuming that 25% of heat is absorbed by the obstacle, then the velocity of the bullet at the time of striking (M.P. of lead = 327°*C*, specific heat of lead = 0.03 *cal*/*gm*°*C*, latent heat of fusion of lead = 6 *cal*/*gm* and *J* = 4.2 *joule*/*cal*)

(A) 410 *m*/*sec* (B) 1230 *m*/*sec* (C) 307.5 *m*/*sec* (D) None of the above

3. The temperature of equal masses of three different liquids *A*, *B* and *C* are 12º*C*, 19º*C* and 28º*C* respectively. The temperature when *A* and *B* are mixed is 16º*C* and when *B* and *C* are mixed is 23º*C*. The temperature when *A* and *C* are mixed is

(A) 18.2º*C* (B) 22º*C* (C) 20.2º*C* (D) 25.2º*C*

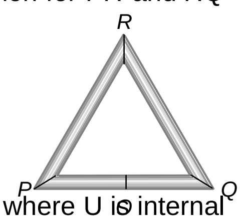
4. In a vertical U-tube containing a liquid, the two arms are maintained at different temperatures $t_1$ and $t_2$. The liquid columns in the two arms have heights $l_1$ and $l_2$ respectively. The coefficient of volume expansion of the liquid is equal to

(A) $\frac{l_1 - l_2}{l_2 t_1 - l_1 t_2}$ (B) $\frac{l_1 - l_2}{l_1 t_1 - l_2 t_2}$ (C) $\frac{l_1 + l_2}{l_2 t_1 + l_1 t_2}$ (D) $\frac{l_1 + l_2}{l_1 t_1 + l_2 t_2}$

5. The coefficient of linear expansion of crystal in one direction is $\alpha_1$ and that in every direction perpendicular to it is $\alpha_2$. The coefficient of cubical expansion is

(A) $\alpha_1 + \alpha_2$ (B) $2\alpha_1 + \alpha_2$ (C) $\alpha_1 + 2\alpha_2$ (D) None of these

6. Three rods of equal length *l* are joined to form an equilateral triangle *PQR*. *O* is the mid point of *PQ*. Distance *OR* remains same for small change in temperature. Coefficient of linear expansion for *PR* and *RQ* is same *i.e.* $\alpha_2$ but that for *PQ*is $\alpha_1$. Then



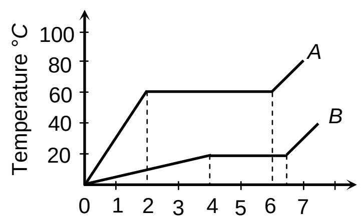

(A) $\alpha_2 = 3\alpha_1$ (B) $\alpha_2 = 4\alpha_1$

(C) $\alpha_1 = 3\alpha_2$ (D) $\alpha_1 = 4\alpha_2$

7. A certain amount of ideal mono atomic gas undergoes, process given by $UV^{1/2} = \text{const}$, where U is internal energy of the gas. Specific heat of the gas in the process is ______

(A) R/2 (B) 3R (C) SR/2 (D) –R/2

8. Two substances *A* and *B* of equal mass *m* are heated at uniform rate of 6 *cal s*$^{-1}$ under similar conditions. A graph between temperature and time is shown in figure. Ratio of heat absorbed $H_A/H_B$ by them for complete fusion is




(A) $\frac{9}{4}$ (B) $\frac{4}{9}$

(C) $\frac{8}{5}$ (D) $\frac{5}{8}$

9. A steel meter scale is to be ruled so that millimeter intervals are accurate within about $5 \times 10^{-5} mm$ at a certain temperature. The maximum temperature variation allowable during the ruling is (Coefficient of linear expansion of steel $= 10 \times 10^{-6} K^{-1}$)

(A) 2°$C$ (B) 5°$C$ (C) 7°$C$ (D) 10°$C$

10. If earth suddenly stops rotating about its own axis, the increase in it's temperature will be

(A) $\frac{R^2\omega^2}{5Js}$ (B) $\frac{R^2\omega^2}{Js}$ (C) $\frac{Rm\omega^2}{5Js}$ (D) None of these

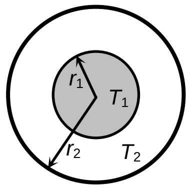
11. There is formation of layer of snow $x cm$ thick on water, when the temperature of air is $-\theta^o C$ (less than freezing point). The thickness of layer increases from $x$ to $y$ in the time $t$, then the value of $t$ is given by

(A) $\frac{(x+y)(x-y)\rho L}{2k\theta}$ (B) $\frac{(x-y)\rho L}{2k\theta}$ (C) $\frac{(x+y)(x-y)\rho L}{k\theta}$ (D) $\frac{(x-y)\rho L k}{2\theta}$

12. A body initially at 80$^oC$ cools to 64$^oC$ in 5 *minutes* and to 52$^oC$ in 10 *minutes*. The temperature of the body after 15 *minutes* will be

(A) 42.7 $^oC$ (B) 35 $^oC$ (C) 47 $^oC$ (D) 40 $^oC$

13. The figure shows a system of two concentric spheres of radii $r_1$ and $r_2$ and kept at temperatures $T_1$ and $T_2$, respectively. The radial rate of flow of heat in a substance between the two concentric spheres is proportional to




(A) $\frac{r_1 r_2}{(r_1 - r_2)}$ (B) $(r_2 - r_1)$

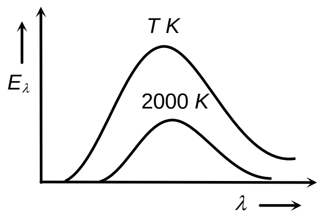
(C) $(r_2 - r_1)(r_1 r_2)$ (D) ln $\left(\frac{r_2}{r_1}\right)$

14. Four rods of identical cross-sectional area and made from the same metal form the sides of square. The temperature of two diagonally opposite points and $T$ and $\sqrt{2}\,T$ respective in the steady state. Assuming that only heat conduction takes place, what will be the temperature difference between other two points

(A) $\frac{\sqrt{2}+1}{2}T$ (B) $\frac{2}{\sqrt{2}+1}T$ (C) 0 (D) None of these

15. The adjoining diagram shows the spectral energy density distribution $E_\lambda$ of a black body at two different temperatures. If the areas under the curves are in the ratio 16 : 1, the value of temperature $T$ is

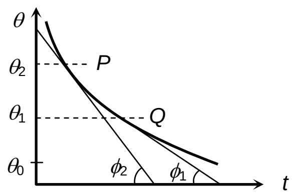
(A) 32,000 $K$
(B) 16,000 $K$
(C) 8,000 $K$
(D) 4,000 $K$



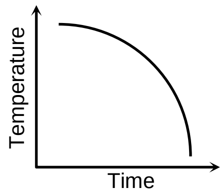

16. A body cools in a surrounding which is at a constant temperature of $\theta_0$. Assume that it obeys Newton's law of cooling. Its temperature $\theta$ is plotted against time $t$. Tangents are drawn to the curve at the points $P(\theta = \theta_1)$ and $Q(\theta = \theta_2)$. These tangents meet the time axis at angles of $\phi_2$ and $\phi_1$, as shown

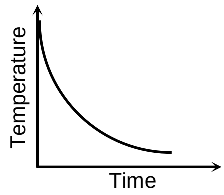
(A) $\frac{\tan\phi_2}{\tan\phi_1} = \frac{\theta_1 - \theta_0}{\theta_2 - \theta_0}$ (B) $\frac{\tan\phi_2}{\tan\phi_1} = \frac{\theta_2 - \theta_0}{\theta_1 - \theta_0}$

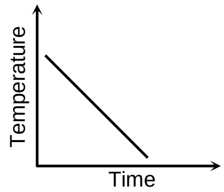
(C) $\frac{\tan\phi_1}{\tan\phi_2} = \frac{\theta_1}{\theta_2}$ (D) $\frac{\tan\phi_1}{\tan\phi_2} = \frac{\theta_2}{\theta_1}$

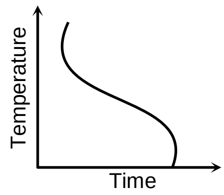


17. A block of metal is heated to a temperature much higher than the room temperature and allowed to cool in a room free from air currents. Which of the following curves correctly represents the rate of cooling

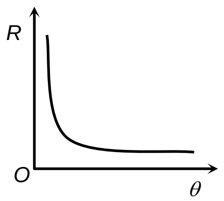
(A) 

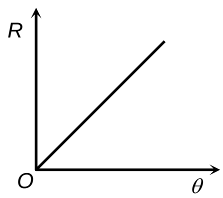

(B) 

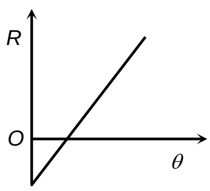

(C) 

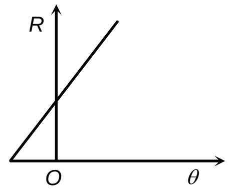

(D) 


18. For a small temperature difference between the body and the surroundings the relation between the rate of loss heat $R$ and the temperature of the body is depicted by

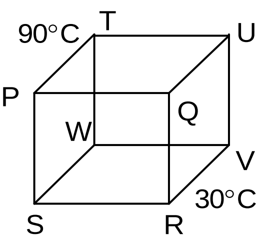
(A) 


(B) 


(C) 


(D) 


19. 12 identical rods are arranged in the form of a cube. The temperatures of P and R are maintained at $90°C$ and $30°C$ respectively. Find the steady state temperature of V.




(A) 65°C (B) 60°C
(C) 20°C (D) 50°C

20. There is a small source of heat radiations emitting energy at a constant rate of P watts. A disc of mass M, area A and specific heat capacity C is placed at a distance 'r' from it. The time taken by disc for the raise in temperature by T Kelvin is ________

(A) $\frac{MCT}{P}$ (B) $\frac{4\pi r^2 MCT}{PA}$ (C) $\frac{\pi r^2 MCT}{PA}$ (D) Data insufficient

**Numerical Based:**

21. The coefficient of apparent expansion of mercury in a glass vessel is $153 \times 10^{-6}/^oC$ and in a steel vessel is $144 \times 10^{-6}/^oC$. If $\alpha$ for steel is $12 \times 10^{-6}/^oC$, then that of glass is ________ in $\times 10^{-6}/^oc$

22. A glass flask of volume one *litre* at 0°$C$ is filled, level full of mercury at this temperature. The flask and mercury are now heated to 100°$C$. How much mercury will spill out, if coefficient of volume expansion of mercury is $1.82\times10^{-4}/°C$ and linear expansion of glass is $0.1\times10^{-4}/°C$ respectively ________ cc.

23. A steel scale measures the length of a copper wire as $80.0\,cm$, when both are at $20°C$ (the calibration temperature for scale). What would be the scale read for the length of the wire when both are at $40°C$? (Given $\alpha_{\text{steel}}=11\times10^{-6}$ per°$C$ and $\alpha_{\text{copper}}=17\times10^{-6}\,per\,°C$ )

24. A piece of metal weight 46*gm* in air, when it is immersed in the liquid of specific gravity 1.24 at 27ºC it weighs 30 *gm*. When the temperature of liquid is raised to 42ºC the metal piece weight 30.5 *gm*, specific gravity of the liquid at 42ºC is 1.20, then the linear expansion of the metal will be _______ × $10^{-5}$/ºc

25. A one *litre* glass flask contains some mercury. It is found that at different temperatures the volume of air inside the flak remains the same. What is the volume of mercury in this flask if coefficient of linear expansion of glass is $9\times10^{-6}/°C$ while of volume expansion of mercury is $1.8\times10^{-4}/°C$

**KEY**

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1. | B | 2. | A | 3. | C | 4. | A | 5. | C |
| 6. | D | 7. | D | 8. | C | 9. | B | 10. | A |
| 11. | A | 12. | A | 13. | A | 14. | C | 15. | D |
| 16. | B | 17 | b | 18. | C | 19. | D | 20. | C |
| 21. | 9 | 22. | 15.2 | 23. | 80.01 | 24. | 2.32 | 25. | 150 |

**SOLUTIONS**

1. Heat lost in $t$ sec = $mL$ or heat lost per sec = $\frac{mL}{t}$. This must be the heat supplied for keeping the substance in molten state per sec.

$\therefore \frac{mL}{t}=P$ or $L=\frac{Pt}{m}$

2. If mass of the bullet is *mgm*,

then total heat required for bullet to just melt down

$Q_1 = m\,c\,\Delta\theta + m\,L = m\times 0.03\,(327-27) + m\times 6$

$= 15\,m\,cal = (15m\times4.2)J$

Now when bullet is stopped by the obstacle, the loss in its mechanical energy $=\frac{1}{2}(m\times10^{-3})v^2 J$

(As $m\,gm = m\times10^{-3}\,kg$)

As 25% of this energy is absorbed by the obstacle,

The energy absorbed by the bullet

$Q_2=\frac{75}{100}\times\frac{1}{2}mv^2\times10^{-3}=\frac{3}{8}mv^2\times10^{-3}J$

Now the bullet will melt if $Q_2\ge Q_1$

*i.e.* $\frac{3}{8}mv^2\times10^{-3}\ge15m\times4.2\Rightarrow v_{\min}=410\,m/s$

3. Heat gain = heat lost

$C_A(16-12) = C_B(19-16)\Rightarrow\frac{C_A}{C_B}=\frac{3}{4}$

and $C_B(23-19) = C_C(28-23)\Rightarrow\frac{C_B}{C_C}=\frac{5}{4}$

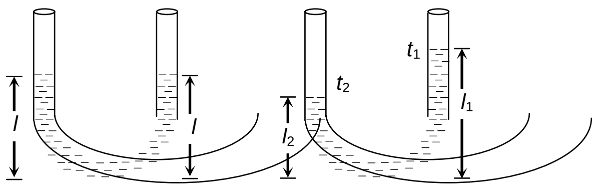
$\Rightarrow \frac{C_A}{C_C} = \frac{15}{16}$ ...(i)

If $\theta$ is the temperature when $A$ and $C$ are mixed then,

$C_A(\theta - 12) = C_C(28 - \theta) \Rightarrow \frac{C_A}{C_C} = \frac{28-\theta}{\theta - 12}$ ...(ii)

On solving equation (i) and (ii) $\theta$= 20.2°$C$

4. Suppose, height of liquid in each arm before rising the temperature is $l$.




With temperature rise height of liquid in each arm increases *i.e.*$l_1 > l$ and $l_2 > l$

Also $l = \frac{l_1}{1+\gamma t_1} = \frac{l_2}{1+\gamma t_2}$

$\Rightarrow l_1 + \gamma l_1 t_2 = l_2 + \gamma l_2 t_1 \Rightarrow \gamma = \frac{l_1 - l_2}{l_2 t_1 - l_1 t_2}$

5. $V = V_0(1+\gamma\Delta\theta)$

$L^3 = L_0(1+\alpha_1\Delta\theta)L_0^2(1+\alpha_2\Delta\theta)^2 = L_0^3(1+\alpha_1\Delta\theta)(1+\alpha_2\Delta\theta)^2$

Since $L_0^3 = V_0$ and $L^3 = V$

Hence $1+\gamma\Delta\theta = (1+\alpha_1\Delta\theta)(1+\alpha_2\Delta\theta)^2$

$\cong (1+\alpha_1\Delta\theta)(1+2\alpha_2\Delta\theta) \cong (1+\alpha_1\Delta\theta + 2\alpha_2\Delta\theta)$

$\Rightarrow \gamma = \alpha_1 + 2\alpha_2$

6. $(OR)^2 = (PR)^2 - (PO)^2 = l^2 - \left(\frac{l}{2}\right)^2$

$= [l(1+\alpha_2 t)]^2 - \left[\frac{l}{2}(1+\alpha_1 t)\right]^2$

$l^2 - \frac{l^2}{4} = l^2(1+\alpha_2^2 t^2 + 2\alpha_2 t) - \frac{l^2}{4}(1+\alpha_1^2 t^2 + 2\alpha_1 t)$

Neglecting $\alpha_2^2 t^2$ and $\alpha_1^2 t^2$

$0 = l^2(2\alpha_2 t) - \frac{l^2}{4}(2\alpha_1 t) \Rightarrow 2\alpha_2 = \frac{2\alpha_1}{4} \Rightarrow; \alpha_1 = 4\alpha_2$

7. $TV^{1/2} = \text{const} \Rightarrow PV^{3/2} = \text{const}$

$C = C_v + \frac{R}{1-\frac{3}{2}} = -\frac{R}{2}$

8. From given curve,

Melting point for $A = 60°C$

and melting point for $B = 20°C$

Time taken by $A$ for fusion $= (6-2) = 4$ minute

Time taken by $B$ for fusion $= (6.5-4) = 2.5$ minute

Then $\frac{H_A}{H_B} = \frac{6\times 4\times 60}{6\times 2.5\times 60} = \frac{8}{5}$

9. As we know $\alpha = \frac{\Delta L}{L_0\Delta\theta} \Rightarrow \Delta\theta = \frac{\Delta L}{\alpha L_0} = \frac{5\times 10^{-5}}{10\times 10^{-6}\times 1} = 5°C$

10. $W = JQ \Rightarrow \frac{1}{2}I\omega^2 = J(MS\Delta\theta) \Rightarrow \frac{1}{2}\left(\frac{2}{5}MR^2\right)\omega^2$

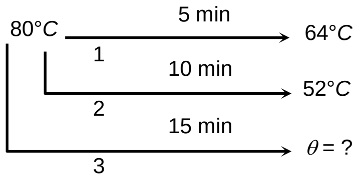
$= J(MS\Delta\theta) \Rightarrow \Delta\theta = \dfrac{R^2\omega^2}{5Js}$

11. Since $t = \dfrac{\rho L}{2k\theta}(x_2^2 - x_1^2)$

$\therefore\ t = \dfrac{\rho L}{2k\theta}(x^2 - y^2) = \dfrac{\rho L(x+y)(x-y)}{2K\theta}$

12. According to Newton law of cooling

$$\frac{\theta_1 - \theta_2}{t} = K\left[\frac{\theta_1 + \theta_2}{2} - \theta_0\right]$$



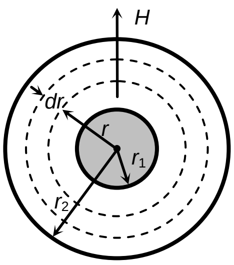

For first process : $\dfrac{(80-64)}{5} = K\left[\dfrac{80+64}{2} - \theta_0\right]$ ...(i)

For second process : $\dfrac{(80-52)}{10} = K\left[\dfrac{80+52}{2} - \theta_0\right]$ ...(ii)

For third process : $\dfrac{(80-\theta)}{15} = K\left[\dfrac{80+\theta}{2} - \theta_0\right]$ …(iii)

On solving equation (i) and (ii) we get $K = \dfrac{1}{15}$ and $\theta_0 = 24°C$. Putting these values in equation (iii) we get $\theta = 42.7°C$

13. Consider a concentric spherical shell of radius *r* and thickness *dr* as shown in fig.




The radial rate of flow of heat through this shell in steady state will be $H = \dfrac{dQ}{dt} = -KA\dfrac{dT}{dr} = -K(4\pi r^2)\dfrac{dT}{dr}$

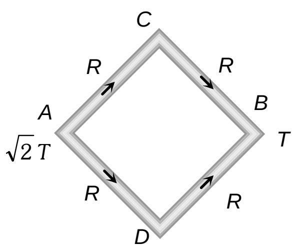
$\Rightarrow \displaystyle\int_{r_1}^{r_2}\frac{dr}{r^2} = -\frac{4\pi K}{H}\int_{T_1}^{T_1} dT$

Which on integration and simplification gives

$H = \dfrac{dQ}{dt} = \dfrac{4\pi K r_1 r_2 (T_1 - T_2)}{r_2 - r_1} \Rightarrow \dfrac{dQ}{dt} \propto \dfrac{r_1 r_2}{(r_2 - r_1)}$

14. Similar to Q.No.26

Temperature difference between *C* and *D* is zero.




15. $\dfrac{A_T}{A_{2000}} = \dfrac{16}{1}$ (given)

Area under $e_\lambda - \lambda$ curve represents the emissive power of body and emissive power $\propto T^4$

(Hence area under $e_\lambda - \lambda$ curve) $\propto T^4$

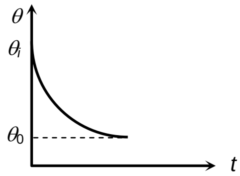
$\Rightarrow \dfrac{AT}{A_{2000}} = \left(\dfrac{T}{2000}\right)^4 \Rightarrow \dfrac{16}{1} = \left(\dfrac{T}{2000}\right)^4 \Rightarrow T = 4000\,K.$

16. For $\theta$-$t$ plot, rate of cooling $= \frac{d\theta}{dt} =$ slope of the curve.

At $P$, $\frac{d\theta}{dt} = \tan\phi_2 = k(\theta_2 - \theta_0)$, where $k$ = constant.

At $Q$ $\frac{d\theta}{dt} = \tan\phi_1 = k(\theta_1 - \theta_0) \Rightarrow \frac{\tan\phi_2}{\tan\phi_1} = \frac{\theta_2 - \theta_0}{\theta_1 - \theta_0}$

17. According to Newton's law of cooling




Rate of cooling $\propto$ Temperature difference

$\Rightarrow -\frac{d\theta}{dt} \propto (\theta - \theta_0) \Rightarrow -\frac{d\theta}{dt} = \alpha(\theta - \theta_0)$ ($\alpha$= constant)

$\Rightarrow \int_{\theta_i}^{\theta} \frac{d\theta}{(\theta - \theta_0)} = -\alpha \int_0^t dt \Rightarrow \theta = \theta_0 + (\theta_i - \theta_0)e^{-\alpha t}$

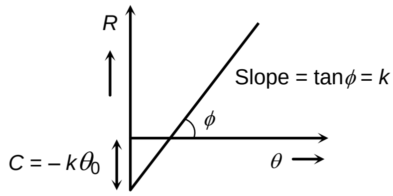
This relation tells us that, temperature of the body varies exponentially with time from $\theta_i$ to $\theta_0$

Hence graph (*b*) is correct

18. Rate of loss of heat ($R$) $\propto$ temperature difference

$\Rightarrow R \propto (\theta - \theta_0) \Rightarrow R = k(\theta - \theta_0) = k\theta - k\theta_0$ ($k$= constant)

on comparing it with $y = mx + c$ it is observed that, the graph between $R$ and $\theta$ will be straight line with slope $=k$ and intercept $= -k\theta_0$




19. Apply K.V.L & K.C.L

20. $I \times A = \text{Power}$; $\text{Power} \times t = \text{Heat}$

$\therefore t = \frac{\text{Heat}}{\text{Power}} = \frac{MCT4\pi r^2}{PA}$

21. $\gamma_{real} = \gamma_{app.} + \gamma_{vessel}$

So $(\gamma_{app.} + \gamma_{vessel})_{glass} = (\gamma_{app.} + \gamma_{vessel})_{steel}$

$\Rightarrow 153 \times 10^{-6} + (\gamma_{vessel})_{glass} = (144 \times 10^{-6} + \gamma_{vessel})_{steel}$

Further, $(\gamma_{vessel})_{steel} = 3\alpha = 3 \times (12 \times 10^{-6}) = 36 \times 10^{-6}/^oC$

$\Rightarrow 153 \times 10^{-6} + (\gamma_{vessel})_{glass} = 144 \times 10^{-6} + 36 \times 10^{-6}$

$\Rightarrow (\gamma_{vessel})_{glass} = 3\alpha = 27 \times 10^{-6}/^oC \Rightarrow \alpha = 9 \times 10^{-6}/^oC$

22. Due to volume expansion of both liquid and vessel, the change in volume of liquid relative to container is given by $\Delta V = V_0[\gamma_L - \gamma_g]\Delta\theta$

Given $V_0 = 1000\ cc$, $\alpha_g = 0.1 \times 10^{-4}/°C$

$\therefore \gamma_g = 3\alpha_g = 3 \times 0.1 \times 10^{-4}/°C = 0.3 \times 10^{-4}/°C$

$\therefore \Delta V = 1000\,[1.82 \times 10^{-4} - 0.3 \times 10^{-4}] \times 100 = 15.2\ cc$

**23.** With temperature rise (same 25°$C$ for both), steel scale and copper wire both expand. Hence length of copper wire *w.r.t.* steel scale or apparent length of copper wire after rise in temperature

$L_{app} = L'_{cu} - L'_{steel} = [L_0(1 + \alpha_{Cu}\Delta\theta) - L_0(1 + \alpha_s\Delta\theta)$

$\Rightarrow L_{app} = L_0(\alpha_{Cu} - \alpha_s)\Delta\theta$

$= 80(17 \times 10^{-6} - 11 \times 10^{-6}) \times 20 = 80.0096\ cm$

24. Loss of weight at 27°$C$ is

$= 46 - 30 = 16 = V_1 \times 1.24\ \rho_l \times g$ ...(i)

Loss of weight at 42°$C$ is

$= 46 - 30.5 = 15.5 = V_2 \times 1.2\ \rho_l \times g$ ...(ii)

Now dividing (i) by (ii), we get $\frac{16}{15.5} = \frac{V_1}{V_2} \times \frac{1.24}{1.2}$

But $\frac{V_2}{V_1} = 1 + 3\alpha\,(t_2 - t_1) = \frac{15.5 \times 1.24}{16 \times 1.2} = 1.001042$

$\Rightarrow 3\alpha\,(42^o - 27^o) = 0.001042 \Rightarrow \alpha = 2.316 \times 10^{-5}/^oC$

25. It is given that the volume of air in the flask remains the same. This means that the expansion in volume of the vessel is exactly equal to the volume expansion of mercury.

*i.e.*, $\Delta V_g = \Delta V_L$ or $V_g \gamma_g \Delta\theta = V_L \gamma_L \Delta\theta$

$\therefore V_L = \frac{V_g \gamma_g}{\gamma_L} = \frac{1000 \times (3 \times 9 \times 10^{-6})}{1.8 \times 10^{-4}} = 150cc$